Douglass Houghton Workshop, Section 1, Mon 09/11/23

Worksheet Dragon

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's apparently a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

   Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

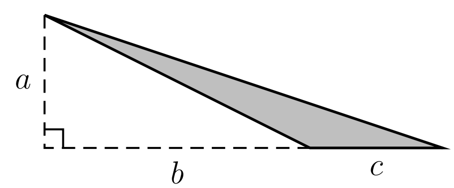
   (a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

   (b) What's the formula for $N(T)$? (We found this previously).

   (c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

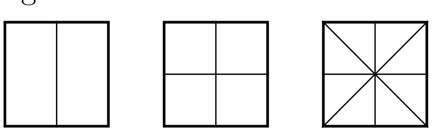
   (d) Verify your guess by finding a formula that fits the data.

   (e) Using the formula we found last time for splitting the towel into n parts, write a limit equation to express the result in part (d).



2. Find the area of the shaded triangle:

3. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. Last time we figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:

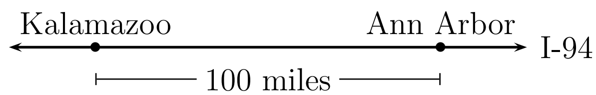


   We had a number of other ideas too. What other numbers of people can you accommodate? Explain exactly how to cut the cake and why it is fair.

4. dBase™ was a database management system popular on IBM PCs back in the 80s, and still used in some places today. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

5. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



(a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

(b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

(c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

6. (This problem appeared on a Winter, 2017 Math 115 exam.) A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.

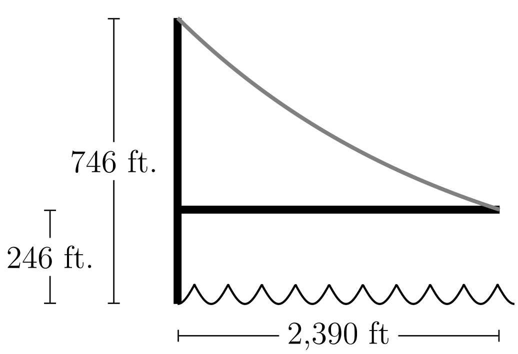
(a) The temperature in Model A goes from its minimum temperature of $-3\,^{\circ}\mathrm{C}$ to its maximum temperature of $15\,^{\circ}\mathrm{C}$ and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is $15\,^{\circ}\mathrm{C}$. Let $A(t)$ be the temperature (in $^{\circ}\mathrm{C}$) inside this chamber t hours after midnight. Find a formula for $A(t)$ assuming it is a sinusoidal function.

(b) Let $B(t)$ be the temperature (in $^{\circ}\mathrm{C}$) inside Model B t hours after midnight, where

$$B(t) = 5 - 3\cos\left(\frac{3}{7}t + 1\right).$$

Find the two smallest positive values of t at which the temperature in the chamber is $6\,^{\circ}\mathrm{C}$. Your answer must be found algebraically. *Show all your work and give your answers in exact form.*

7. (This question appeared on a Fall, 2008 Math 115 exam.) San Francisco's famous Golden Gate bridge has two towers which stand 746 ft. above the water, while the bridge itself is only 246 ft. above the water. The last leg of the bridge, which connects to Marin County, is 2,390 ft. long. The suspension cables connecting the top of the tower to the mainland can be modeled by an exponential function. Let $H(x)$ be the function describing the height above the water of the suspension cable as a function of x, the horizontal distance from the tower.



(a) Find a formula for $H(x)$.

(b) The engineers determined that some repairs are necessary to the suspension cables. They climb up the tower to 400 ft above the bridge, and they need to lay a horizontal walking board between the tower and the suspension cable. How long does the walking board need to be to reach the cable?