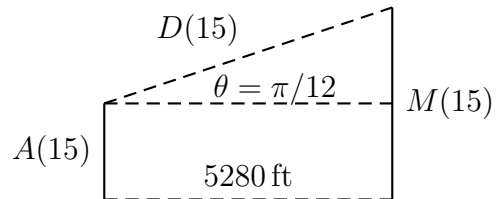


## Worksheet To Infinity, and Beyond

1. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Myrka takes the train to the North Pole. It's the Polar Express, of course, and her plan is to steal one of Santa's reindeer. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Myrka spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let  $M(t)$  be the distance between the car and its starting point, and  $A(t)$  be Myrka's distance from her starting point. After watching the car for 15 seconds, Myrka has rotated her head  $\pi/12$  radians.

- (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Myrka and the car 15 seconds after she starts watching it.



- (b) Let  $\theta(t)$  be the angle Myrka has turned her head after tracking the car for  $t$  seconds. Write an equation for the distance between Myrka and the car at time  $t$ . (Your answer may involve  $\theta(t)$ .)
- (c) If at precisely 15 seconds, Myrka is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Myrka and the car?
- (d) What is the speed of the car at 15 seconds?
2. Suela is studying a colony of *campylobacter jejuni* bacteria. She finds that the growth rate of the colony is increasing exponentially. That is, if  $P(t)$  is the population in thousands after  $t$  hours, then  $P'(t) = Ae^{kt}$  for some constants  $A$  and  $k$ .
- (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after  $T$  hours.
- (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after  $T$  hours.
- (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants  $A$  and  $k$ .
- (d) How many bacteria are there 6 hours after the experiment started?
3. Suela does another experiment with the same starting population of bacteria, but this time she plays Bazzi to the bacteria. She finds they LOVE Bazzi, and they grow while a song is playing and stop growing between songs. She plays them a series of 3-minute songs with 3-minute breaks between them, and finds that  $t$  hours after the experiment starts, their growth rate (in thousands per hour) is  $1 + \sin(20\pi t)$ . How many bacteria grow in each 6-minute cycle?