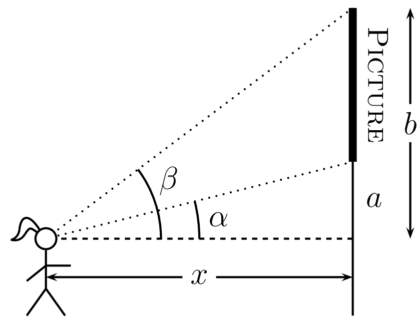
Douglass Houghton Workshop, Section 2, Tue 12/03/19

Worksheet Romeo, Romeo, Wherefore Art Thou Romeo?

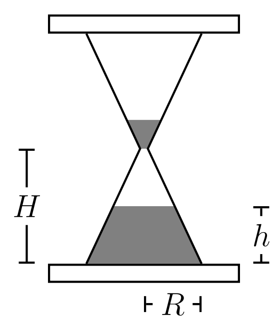
1. Suppose a picture is mounted on the wall. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

   (a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

   (b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.



2. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



3. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

   (a) $P'(2) = 55$

   (b) $\displaystyle\int_5^{10} P'(t)\,dt$

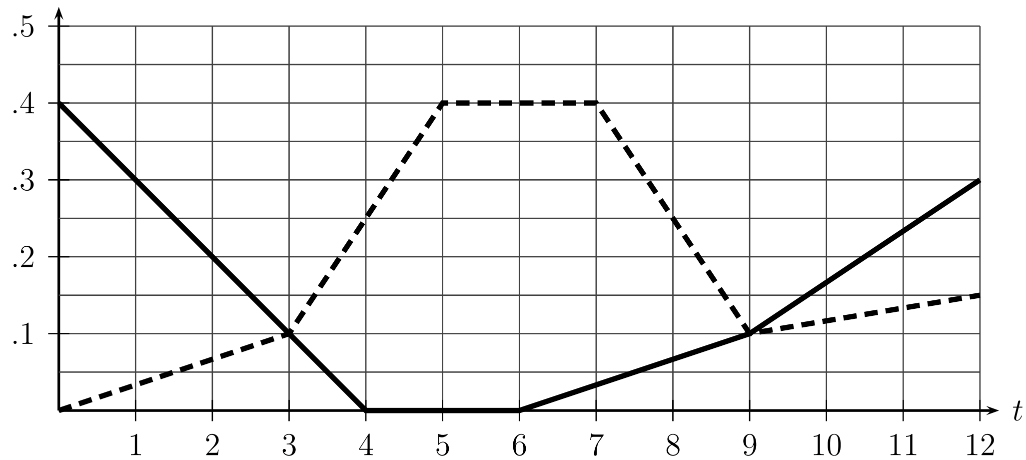
   (c) $P^{-1}(690)$

   (d) $\displaystyle\frac{1}{5}\int_5^{10} P(t)\,dt$

4. (This problem appeared on a Fall, 2005 Math 115 Final Exam) Using techniques from calculus, find the dimensions which will maximize the surface area of a circular cylinder whose height h and radius r, each in centimeters, are related by

$$h = 8 - \frac{r^2}{3}.$$

5. (Based on a Fall, 2011 Math 115 Exam Problem) Because Lexi likes all kinds of music except New Country, she gets a job in the music section of the UM library. One day she is working a 12-hour shift, re-shelving vinyl records. The rate $\ell(t)$ (in thousands of records per hour) at which she is adding records to the shelves is represented by the dashed line below in the graph. At the same time, patrons are taking records off the shelves to listen to. The removal rate, $r(t)$, is represented by the solid line. At the beginning of the shift there were 18000 records on the shelves.



(a) Over what period(s) was Lexi gaining ground on the patrons?

(b) When was the number of records on the shelves increasing the fastest? When was it decreasing the fastest?

(c) When was the the number of records on the shelves the greatest? Explain.

(d) How many records were on the shelves at the end of the shift (at $t = 12$)?

(e) Draw a graph of the number of records on the shelves as a function of time. Label all critical points and inflection points.

6. (Fall 2008) This problem was a smörgåsbord:

(a) If $f(x)$ is even and $\displaystyle\int_{-2}^{2} (f(-x) - 3)\,dx = 8$, find $\displaystyle\int_0^2 f(x)\,dx$.

(b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

(c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\displaystyle\int_3^{27} R(t)\,dt$ means in this context.

(d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

(e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \le 1 \\ 6\sin(\pi(x - 0.5)) + k & \text{if } x > 1 \end{cases}$$