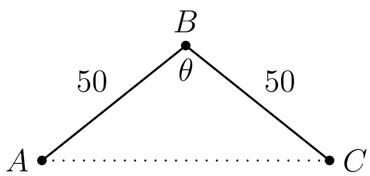
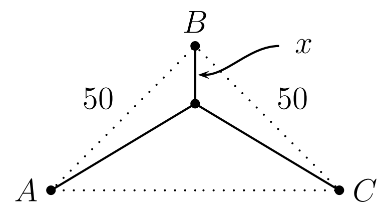
Douglass Houghton Workshop, Section 2, Thu 11/21/19

Worksheet Past is Prologue

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.



   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.

   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Write the following sums in sigma ($\sum$) notation.

   (a) $1 + 2 + 3 + 4 + \cdots 10$

   (b) $1 + 2 + 3 + 4 + \cdots + n$

   (c) $3 + 5 + 7 + 9 + \cdots + 21$

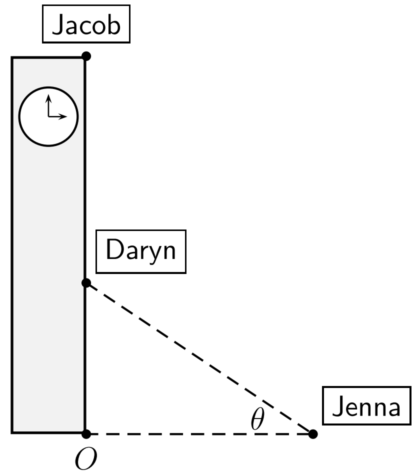
   (d) $4 + 9 + 16 + 25 + \cdots + 100$

   (e) $2.3 + 2.8 + 3.3 + 3.8 + 4.3 + 4.8 + \cdots + 10.3$

   (f) $f(a_1) + f(a_2) + f(a_3) + \cdots + f(a_n)$

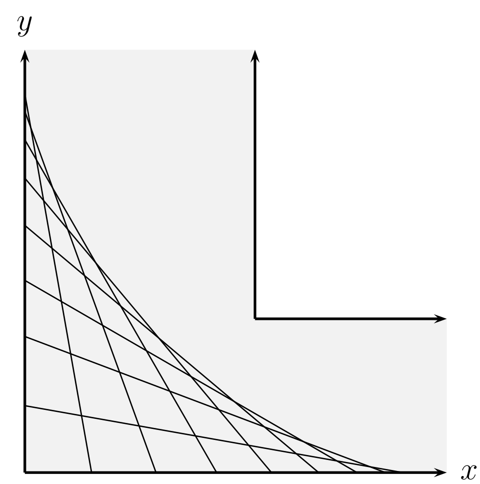
3. (This problem appeared on a Winter, 2004 Math 115 Final Exam.) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$, measured in songs/hour where $t = 0$ corresponds to 5 pm. Explain the meaning of the quantity $\int_0^5 m(t)\,dt$.

4. (Adapted from a Winter, 2005 Math 115 exam) One day Jacob notices that the door to the Burton Tower carillon has been left open. He can't resist the urge to climb to the top of the tower and barricade himselfin. Hethen begins a dramatic one-trombone concert of his favorite tunes from Lord of the Rings .
Because of her basketball experience, the University asks Daryn to scale the tower and retake the carillon. Meanwhile on the ground, 30 feet from the tower, Jenna, on her horse, is performing a stationary dressage routine. She looks up at an angle θ to see Daryn.



   (a) Find the rate of change of Daryn's distance from the point O with respect to θ.
   (b) If the distance from point O to Jacob is 200 ft and Daryn climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Daryn is halfway up?
   (c) When Daryn is halfway up, Jacob drops the end of a rope down to help her. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Daryn catch it, and what is its speed when she does?
   (d) Jenna watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of her gaze changing when Daryn catches the rope?

5. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!
   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

6. Suppose $\int_4^9 (4f(x) + 7)\, dx = 315$. Find $\int_4^9 f(x)\, dx$.