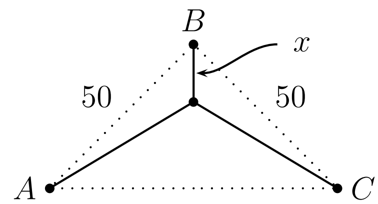
Douglass Houghton Workshop, Section 2, Tue 11/19/19

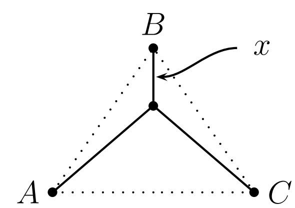
Worksheet Once More, Unto the Breach, Dear Friends

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 50 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 100. But by constructing a ⅄-shaped network like the one at the right, we found

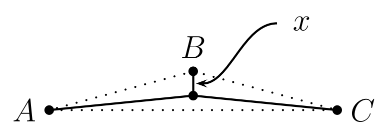


   - The length of the network is $x + 2\sqrt{2500 - 100x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 100$) by increasing x. For instance, when $x = 10$, the network has a length of about 97.



   (a) Consider the case where the triangle is still isosceles and the legs still have length 50, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 100$)?

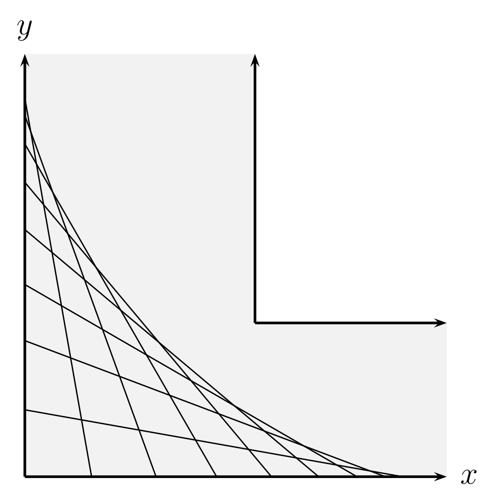


   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

3. (Adapted from a Fall, 2005 Math 115 Final Exam) On Christmas Eve, Antonio makes cookies for Santa. Zita, flying her plane, is trying to beat Santa to the cookies. Assume that Santa is directly north of the Antonio's house (therefore traveling due south) while Zita is directly East of the house (traveling due West—also flying, so as to try to get ahead of Santa). Assume that both Santa and Zita are flying at the same altitude. Santa is moving at 30 miles per hour, and Zita is going 28 miles per hour. How fast is the distance between them changing when Santa is 120 miles from Antonio's house and Zita is 160 miles from the house?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Katie. At the end of every day this month she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1600	7	11700	13	21600	19	31700	25	41600
2	3600	8	13300	14	23300	20	33300	26	43200
3	5200	9	14900	15	25000	21	34900	27	44900
4	6800	10	16600	16	26500	22	36600	28	46600
5	8300	11	18200	17	28200	23	38300	29	48300
6	10100	12	20000	18	30000	24	40000	30	50000

(a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Katie has written at time x. Assume that each day Katie writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

(b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

(c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.

5. (This problem appeared on a Winter, 2016 Math 115 Final Exam.) Consider the family of functions given by $f(x) = e^{x^2+Ax+B}$ for constants A and B.

(a) Find and classify all local extrema of $f(x) = e^{x^2+Ax+B}$. Your answers may depend on A and/or B.

(b) Find the values of A and B that make $(3, 1)$ a critical point of $f(x)$.