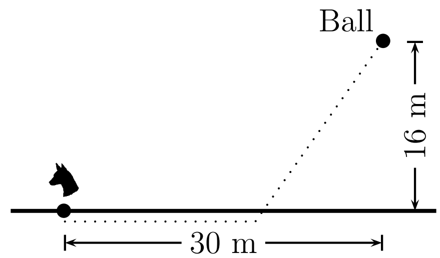
Douglass Houghton Workshop, Section 2, Tue 10/29/19

Worksheet Magnificent

1. Suppose Wade is walking along the shore of Lake Michigan with his dog Paisley. Wade throws a ball 30 meters down the beach and 16 meters out into the water.

   Paisley, being practical, wants to get to the ball as quickly as possible. The thing is that she can run faster than she can swim; her running speed on the beach is 9 meters per second, and she can swim 3 meters per second. How should Paisley (who has an intuitive notion of calculus) get to the ball?



2. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

   points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

   points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

3. Suppose $h(x)$ is a continuous function defined for all real numbers x. The derivative and second derivative of $h(x)$ are given by

$$h'(x) = \frac{2x}{3(x^2-1)^{2/3}} \qquad \text{and} \qquad h''(x) = -\frac{2(x^2+3)}{9(x^2-1)^{5/3}}.$$

   (a) Find the all critical points and local extrema of $h(x)$. Use calculus to classify the critical points and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema.

   (b) Find all inflection points of $h(x)$, and justify that you've found them all.

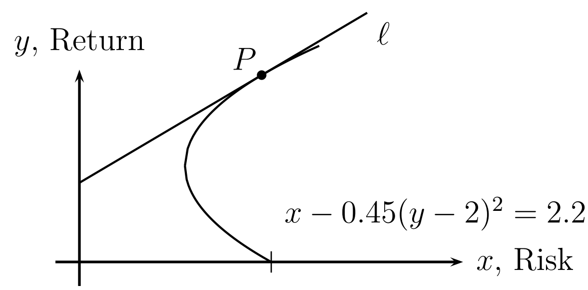
4. (This problem appeared on a Winter, 2008 Math 115 Exam.)

   (a) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f?

   (b) Find all critical points, local maxima, and local minima of f.

   (c) Which of the local maxima and minima are global maxima / minima?

5. (This problem appeared on a Fall, 2008 Math 115 Exam) In Modern Portfolio Theory, a client's portfolio is structured in a way that balances risk and return. For a certain type of portfolio, the risk, x, and return, y, are related by the equation $x - 0.45(y-2)2 = 2.2$. This curve is shown in the graph below. The point P represents a particular portfolio of this type with a risk of 3.8 units. The tangent line, ℓ, through point P is also shown.

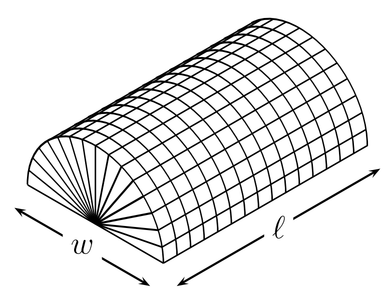


   (a) Using implicit differentiation, find dy/dx, and the coordinates of the point(s) where the slope is undefined.

   (b) The y-intercept of the tangent line for a given portfolio is called the *Risk Free Rate of Return.* Use your answer from (a) to find the Risk Free Rate of Return for this portfolio.

   (c) Now, estimate the return of an optimal portfolio having a risk of 4 units by using your information from part (b). Would this be an overestimate or an underestimate? Why?

6. (From a Winter, 2011 Math 115 exam) A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ, is $1 + 0.001\ell$ dollars per square foot.



   (a) Write an equation for the cost of a hoophouse in terms of ℓ and w.

   (b) Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.