Douglass Houghton Workshop, Section 2, Thu 10/24/19

Worksheet Labradoodle

1. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

   Day 1: A partridge in a pear tree. How to get it down?

   Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

   Day 3: 3 French hens, 2 more turtle doves, and another partidge.

   ...

   Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

   (a) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n's received.

   (b) Of which item does Mr. Truelove send the most? (Solve using calculus.)

2. Mila drives east on the Ohio Turnpike to a robotics competition. She takes a ticket out of the machine in Toledo, and then turns it in at the toll booth near Columbus. Along with her change, the State Trooper in the toll booth hands Mila a speeding citation, and says that he *knows* Mila was going exactly 70 mph at some point on her trip. How does the Mean Value Theorem tell the trooper that?

3. Section 3.8 of your book (which we skip in 115) is about the "hyperbolic trig functions":

   $$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \sinh(x) = \frac{e^x - e^{-x}}{2}$$

   They are often called the even and odd parts of e^x, because they sum to e^x and one is an even function and one is an odd function.

   (a) Which is which?

   (b) Let $f(x)$ be any old function which is defined for all real numbers x. Think of a way to split $f(x)$ into even and odd parts. (Hint: Stare at the definitions above until you get an idea. Then check it.)

   (c) cosh and sinh obey many rules similar, but not exactly the same, as those for cos and sin. To deduce a few, find the derivatives of $\cosh(x)$ and $\sinh(x)$. Then find $\cosh(2x)$ and $\sinh(2x)$. Can you find something resembling $\sin^2 x + \cos^2 x = 1$?

4. (This problem appeared on a Winter, 2009 Math 115 Exam) Suppose a is a positive (non-zero) constant, and consider the function

   $$f(x) = \frac{1}{3}x^3 - 4a^2x.$$

   Determine all maxima and minima of f in the interval $[-3a, 5a]$. For each, specify whether it is global or local.

5. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

   (a) How long should the band play to maximize the number of points Michigan scores?

   (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

   $$B(x) = -x^2 + 8x + 84$$

   points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

   (c) What will be the score of the game for the case you found in part (b)?

6. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

7. (Winter, 2012) Consider the family of functions

$$f(x) = ax - e^{bx},$$

where a and b are positive constants.

   (a) Any function $f(x)$ in this family has only one critical point. In terms of a and b, what are the x- and y-coordinates of that critical point?

   (b) Is the critical point a local maximum or a local minimum? Justify your answer with either the first-derivative test or the second-derivative test.

   (c) For which values of a and b will $f(x)$ have a critical point at $(1, 0)$?