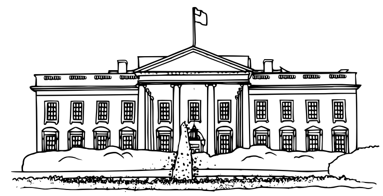
Douglass Houghton Workshop, Section 2, Thu 10/17/19

Worksheet Journey into Night



1. We still have this $1/z$ scale model of the White House, which we plan on blowing up. We want to decide what speed to run the film at, so that when we slow it down to 24 frames per second, we get a realistic explosion.

   (a) Last time we showed that an object will fall $16t^2$ feet in t seconds. So how long does it take for an object to fall off the real white house, which is H feet tall? How many frames will that be, if we film it at 24 frames per second and show it at the same speed?

   (b) How long does it take an object to fall off the top of the model?

   (c) How many frames per second should you film to get the right number of frames to make it look like the model is full-sized?

2. Consider a mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

   (a) Draw the mirror (make it big). What shape is it?

   (b) Draw a light ray travelling leftward along the line $y = -b$, where b is some positive number (making $-b$ negative). At what point P does the ray hit the mirror?

   (c) Find, in terms of b, the slope of the tangent to the mirror at P.

   (d) The *normal* to a curve at a point is the line through that point which is perpendicular to the tangent line. Find the slope of the normal to the mirror at P, and draw both the normal and tangent lines on your graph.

   (e) Suppose a line makes an angle θ with the positive x-axis. What is the slope of the line?

   (f) Let θ be the angle the normal to the mirror at P makes with the light ray $y = -b$. Can you write θ in terms of b? Hint: Use (2d) and (2e).

   To be continued...

3. (From Fall, 2011, Math 115 Exam 2) Let $f(x) = \ln(x)$. Use the table of values below for $g(x)$ and $g'(x)$ to answer the following questions.

   (a) If $F(x) = f(g(x))$, find $F'(4)$.

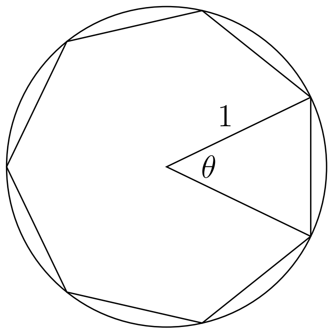
   (b) If $G(x) = g^{-1}(x)$, find $G'(4)$.

   (c) If $H(x) = \tan(g(x))$, find $H'(3)$.

   (d) If $E(x) = e^{f(x)g(x)}$, find $E'(2)$.

x	2	3	4
$g(x)$	1	4	6
$g'(x)$	5	3	2

4. (This problem explains why $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$, but only when θ is measured in radians.) Consider a regular n-sided polygon inscribed in a circle of radius 1.



(a) Let A_n be the area of the polygon. What does A_n approach as n gets large? $\lim_{n\to\infty} A_n = \boxed{}$

(b) We can compute A_n by dividing the polygon up into triangles which have a vertex at the center. Let θ be the vertex angle (in radians). What is θ in terms of n?

(c) What happens to θ as n gets large?

(d) What is the area of one of the triangles, in terms of θ?

(e) What is A_n in terms of θ?

(f) Substitute into the equation from part (a) so that it includes θ's but not n's. Simplify it as much as you can. Hint: $\sin(2x) = 2\sin(x)\cos(x)$.

(g) What would change if we measured θ in degrees instead of radians?

5. (An old team homework problem.) Let $f(x) = x^2 - 2x + 13$ and $g(x) = -x^2 - 2x - 5$.

(a) Draw $y = f(x)$ and $y = g(x)$ on the same set of axes. How many lines are tangent to both graphs?

(b) Find the equations of those lines.

6. (This problem appeared on a Winter 2007 Math 115 exam) Suppose f and g are differentiable functions with values given by the table below.

(a) If $h(x) = f(x)g(x)$, find $h'(3)$.

(b) If $j(x) = \dfrac{(g(x))^3}{f(x)}$, find $j'(1)$.

(c) If $d(x) = x\ln\left(e^{f(x)}\right)$, find $d'(3)$.

(d) If $t(x) = \cos(g(x))$, find $t'(1)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	9	-3	7
3	4	11	15	-19

(d) If $t(x) = \cos(g(x))$, find $t'(1)$.