Douglass Houghton Workshop, Section 2, Thu 09/26/19

Worksheet Harbinger of Things to Come

1. Explain how a slide rule works. This picture may be helpful:

2. (Winter, 2010) Suppose $W(h)$ is an invertible function which tells us how many gallons of water an oak tree of height h uses on a hot summer day.

   (a) Give practical interpretations of $W(50)$ and $W^{-1}(40)$.

   (b) Suppose that an average oak tree is A feet tall and uses G gallons of water on a hot summer day. Answer in terms of W, A, and G:

      i. A farmer has 25 oak trees, and each one is 10 feet taller than an average oak tree. How much water will they use on a hot summer day?

      ii. The farmer also has some oak trees which use 5 fewer gallons of water on a hot summer day than an average oak tree does. How tall are his trees?

3. (This problem appeared on a Winter, 2009 Math 115 Exam) Air pressure, P, decreases exponentially with the height, h, in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

   (a) On top of Mount Denali, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

   (b) Determine $P^{-1}(0.7)$. Include units!

4. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

5. (Winter, 2009) A continuous function f, defined for all x, is always decreasing and concave up. Suppose $f(6) = -6$ and $f'(6) = -1.5$.

   (a) How many zeroes does f have? Justify your answer.

   (b) Can $f'(2) = -1$? Justify your answer.

   (c) Circle all the intervals below in which f has at least one zero. Justify your choices with a picture and a short description.

$$(-\infty, -6) \quad [-6, -2) \quad [-2, -1) \quad [-1, 1) \quad [1, 2) \quad [2, 6) \quad [6, \infty)$$

6. (This problem appeared on a Fall, 2015 Math 115 Exam) Cal is jumping a rope being swung by Gen and Algie while Maddy runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Maddy starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $C(t)$, where t is the number of seconds displayed on Maddy's stopwatch.

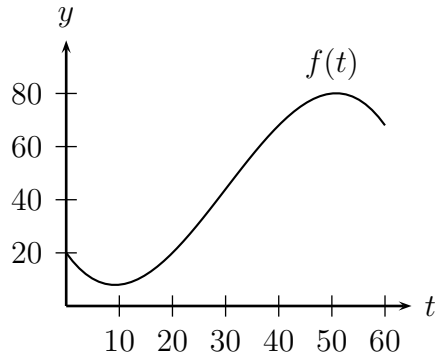
   (a) Sketch a well-labeled graph of two periods of $C(t)$ beginning at $t = 0$.

   (b) Find a formula for $C(t)$.

   (c) Now Gen takes a turn at jumping while Cal and Algie swing the rope. Maddy resets the stopwatch and starts it over again. Let $G(w)$ be the height (in inches above the ground) of the piece of tape when Maddy's stopwatch says w seconds. A formula for $G(w)$ is $G(w) = 41 + 38\cos(2\pi w)$. Maddy is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Maddy's head? (Assume Maddy is standing straight while watching the stopwatch.)

7. (Fall, 2012) Your pet bird is flying in a straight path toward you and away from you for a minute. After t seconds, she is $f(t)$ feet away from you, where

$$f(t) = \frac{-t(t-20)(t-70)}{500} + 20, \qquad 0 \le t \le 60.$$



   (a) Without doing any calculations, determine which is greater: the average velocity of the bird over the entire minute, or her instantaneous velocity after 30 seconds. Explain, referring to the graph.

   (b) Calculate the exact value of the average velocity of the bird over the entire minute.

   (c) Write an explicit expression for the velocity of the bird at time t using the limit definition of velocity. Final answers containing the letter f will receive no credit. Do not evaluate your expression.

   (d) After a minute, you scare the bird, and she flies away at 9 feet/sec. Write a formula for a continuous function $f(t)$ describing the distance between you and the bird for $0 \le t \le 180$.