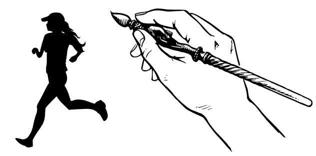
Douglass Houghton Workshop, Section 2, Tue 09/24/19

Worksheet Go Forth and Multiply

1. Sofia has noticed that her tastes changed over the last year. A year ago she spent about 15 hours a week running, and 10 hours painting. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week running and 5 hours a week painting.

   Let $R(t)$ be the number of hours Sofia spent running in week t, and let $P(t)$ be the number of hours she spent painting. Assume $R(t)$ and $P(t)$ are continuous functions of time.

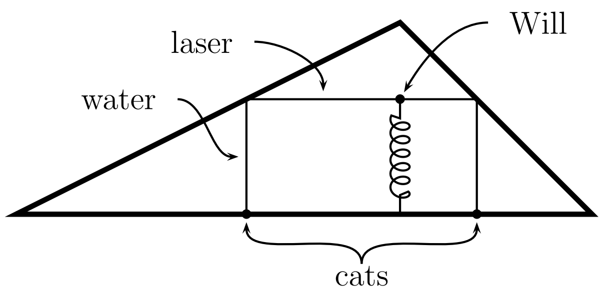
   

   (a) What does it mean for a function to be continuous?

   (b) Are $R(t) + P(t)$, $R(t) - P(t)$, and $R(t)P(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Sofia was spending the same amount of time running and painting.

2. Will is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Will goes up, the cats follow the drops toward the base of the platform.

   (a) Let $h(t)$ be Will's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Will's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Will is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Will's height off the floor.

3. Prove that it's possible to make a fair 5-sided die.

4. Last time we investigated rules for how a population of parakeets might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 200$	800	Yes

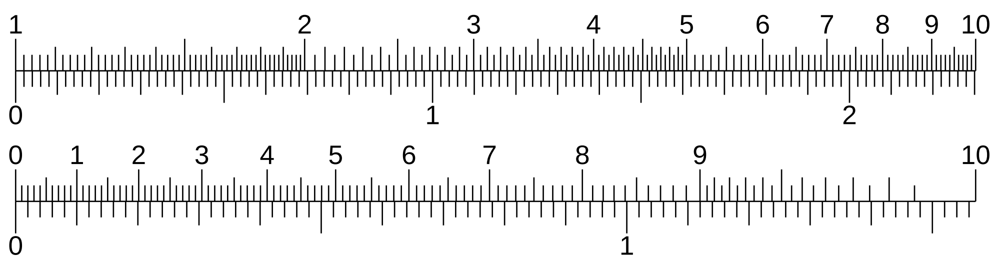
An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

$$P(n+1) = .4P(n) + 600 \qquad P(n+1) = -1.3P(n) + 460$$
$$P(n+1) = 1.1P(n) - 330 \qquad P(n+1) = P(n) + 300$$
$$P(n+1) = -.5P(n) + 1200 \qquad P(n+1) = -P(n) + 300$$

(b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

5. What's the deal with these pictures? What are they good for?



6. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

7. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

(a) What percentage of the pollutant is left after 10 hours?

(b) How long is it before the pollution is reduced by 50%?