Douglass Houghton Workshop, Section 2, Thu 09/19/19

Worksheet Fluffernutter

1. Cake! We saw the outline of a general solution last time. It involoved having 19 triangles along each edge, giving each person 4 of these, and then splitting up the inside in some fashion.

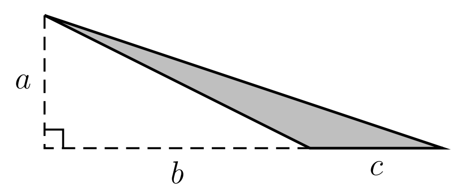
   We need to make this precise, so we can actually cut the cake at the end of class. Suggestions

   (a) Can you make the edge triangles bigger, so that they fill up the entire cake, but still remain equal in area? (Hint: See the next problem.)

   (b) Can you make it so everyone gets only one piece?

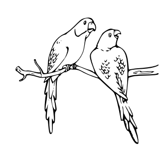
   We want a general solution, but for today, a solution for 10 people will do.

2. Find the area of the shaded triangle:



3. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

4. Cristal is studying a population of parakeets in Australia. Suppose that the population changes according to the rule:



$$P(n+1) = 1.5P(n) - 200$$

   where $P(0)$ is the population in 2019, $P(1)$ is the population 1 year later, etc. (P is measured in parakeets.)

   (a) Make up a (short) story about parakeets that yields that formula as the result.

   (b) Suppose $P = 320$ in 2019. What will happen in the long run?

   (c) Suppose instead that $P = 800$ in 2019. Now what happens?

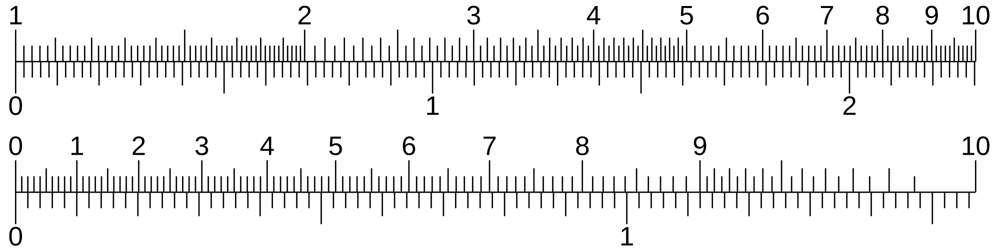
   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

5. Repeat the last problem, but for the rule

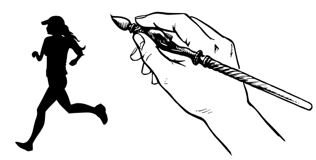
$$P(n+1) = .75P(n) + 125.$$

6. What's the deal with these pictures? What are they good for?



7. Sofia has noticed that her tastes changed over the last year. A year ago she spent about 15 hours a week running, and 10 hours painting. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week running and 5 hours a week painting.

   Let $R(t)$ be the number of hours Sofia spent running in week t, and let $P(t)$ be the number of hours she spent painting. Assume $R(t)$ and $P(t)$ are continuous functions of time.

   (a) What does it mean for a function to be continuous?

   (b) Are $R(t) + P(t)$, $R(t) - P(t)$, and $R(t)P(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Sofia was spending the same amount of time running and painting.