Douglass Houghton Workshop, Section 2, Tue 09/17/19

Worksheet Elephant

1. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head & Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

2. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{T}{n}\right)^n = e^T.$$

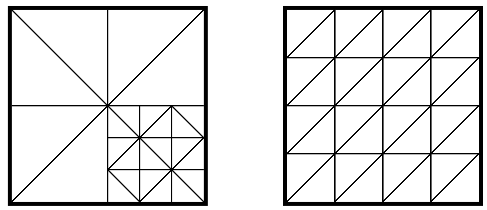
   (b) Devise a similar rule for the time it takes your money to triple at r% interest.

3. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

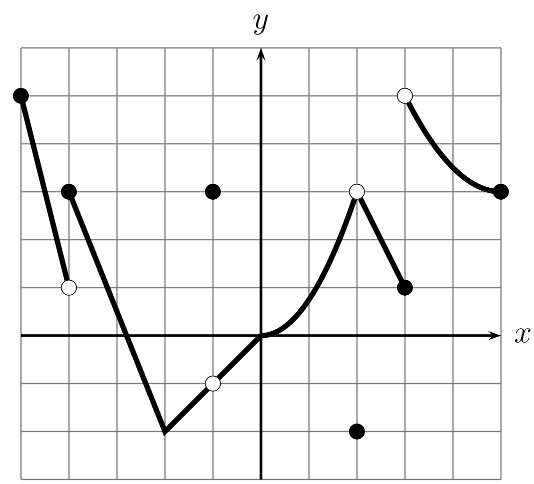
4. Cake! We had a breakthrough last time. We found that we could cut the cake up into lots of little triangles, and give each person the right number of them, making sure everyone gets the same amount of edge frosting. That worked for 6, 16, and 32 people.



We want a general solution for n people, and especially solutions for 19 and 20 people.

5. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

6. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $$[-4,-1] \quad [-2,3] \quad [2,5] \quad [-2,2]$$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim_{x\to -1} Q(x)$ ii. $\lim_{w\to 2} Q(Q(w))$

   iii. $\lim_{h\to 0} \dfrac{Q(-3+h)-Q(-3)}{h}$ iv. $\lim_{x\to\infty} Q\left(\dfrac{1}{x}+3\right)$ v. $\lim_{x\to\frac{1}{3}} xQ(3x-5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim_{x\to p^-} Q(x) \neq Q(p)$?

7. (This problem appeared on a Winter, 2009 Math 115 Exam) Air pressure, P, decreases exponentially with the height, h, in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

   (a) On top of Mount Denali, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

   (b) Determine $P^{-1}(0.7)$. Include units!