Douglass Houghton Workshop, Section 1, Wed 12/11/19

Worksheet Remember What Peace There May Be in Silence

1. Laci is studying a colony of *campylobacter jejuni* bacteria. She finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.

   (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.

   (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.

   (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.

   (d) How many bacteria are there 6 hours after the experiment started?

2. Jennifer also does an experiment with the same starting population of bacteria, but she plays Los Tigres del Norte to the bacteria. She finds they LOVE Los Tigres del Norte, and they grow while a song is playing and stop growing between songs. She plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, their growth rate (in thousands per hour) is $1 + \sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?

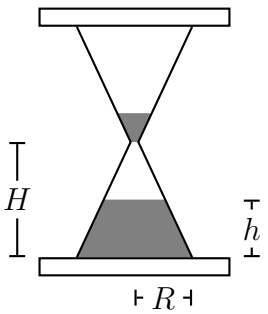
3. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:
$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$

   (a) Find the zeroes of f in terms of A and B.

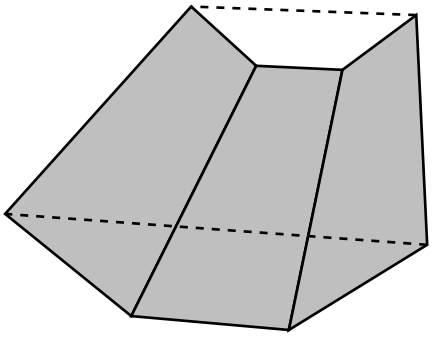
   (b) Find all critical points and inflection points of f in terms of A and B.

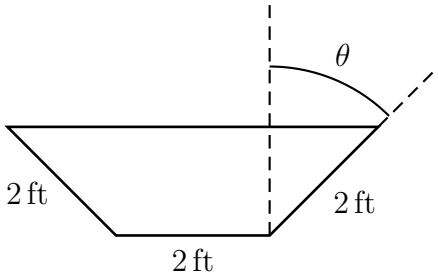
   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

4. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?



5. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

6. (Fall, 2016) Yukiko has a small orchard where she grows Michigan apples. After careful study last season, Yukiko found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a}$$

   for $0 \leq a \leq 320$. Qabil has promised to buy up to 100 bushels of apples for his famous apple ice cream. If Yukiko has any remaining apples, she has an agreement to sell them to Xanthippe's cider mill at a reduced price. Let $R(a)$ be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{if } 0 \leq a \leq 100 \\ 2000 + 50a & \text{if } 100 < a \leq 320 \end{cases}$$

   (a) How much will Xanthippe's cider mill pay per bushel?

   (b) What is Yukiko's fixed cost?

   (c) For what values of a will Yukiko's marginal revenue equal her marginal cost?

   (d) Graph marginal revenue and marginal cost.

   (e) Assuming Yukiko can produce up to 320 bushels of apples, how many bushels should she produce, and what will be the maximum profit?