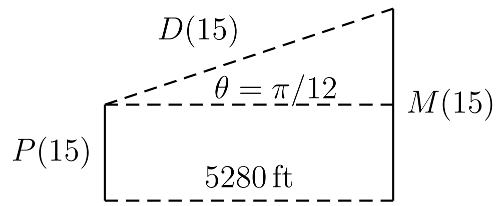
Douglass Houghton Workshop, Section 1, Mon 12/09/19

Worksheet Question Everything

1. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Payton takes the train to Chicago to visit her friend Toby, who is on the Navy base there. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Payton spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let $M(t)$ be the distance between the car and its starting point, and $P(t)$ be Payton's distance from her starting point. After watching the car for 15 seconds, Payton has rotated her head $\pi/12$ radians.

   (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Payton and the car 15 seconds after she starts watching it.

   (b) Let $\theta(t)$ be the angle Payton has turned her head after tracking the car for t seconds. Write an equation for the distance between Payton and the car at time t. (Your answer may involve $\theta(t)$.)

   (c) If at precisely 15 seconds, Payton is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Payton and the car?

   (d) What is the speed of the car at 15 seconds?

2. Laci is studying a colony of *campylobacter jejuni* bacteria. She finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.

   (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.

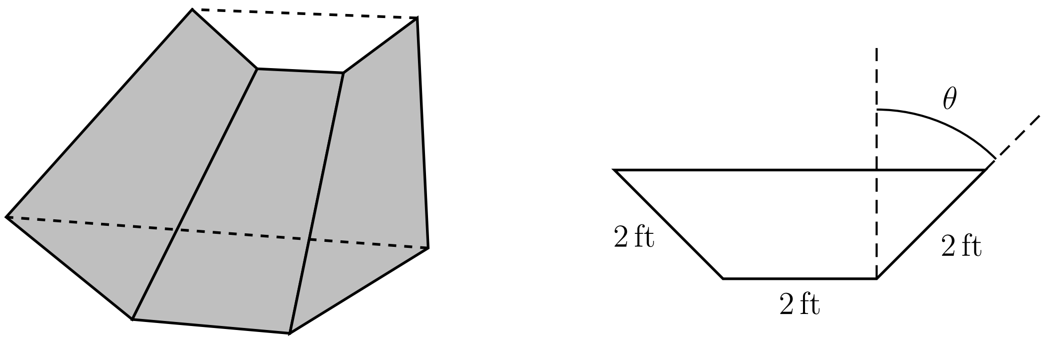
   (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.

   (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.

   (d) How many bacteria are there 6 hours after the experiment started?

3. Jennifer also does an experiment with the same starting population of bacteria, but she plays Los Tigres del Norte to the bacteria. She finds they LOVE Los Tigres del Norte, and they grow while a song is playing and stop growing between songs. She plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, their growth rate (in thousands per hour) is $1 + \sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?

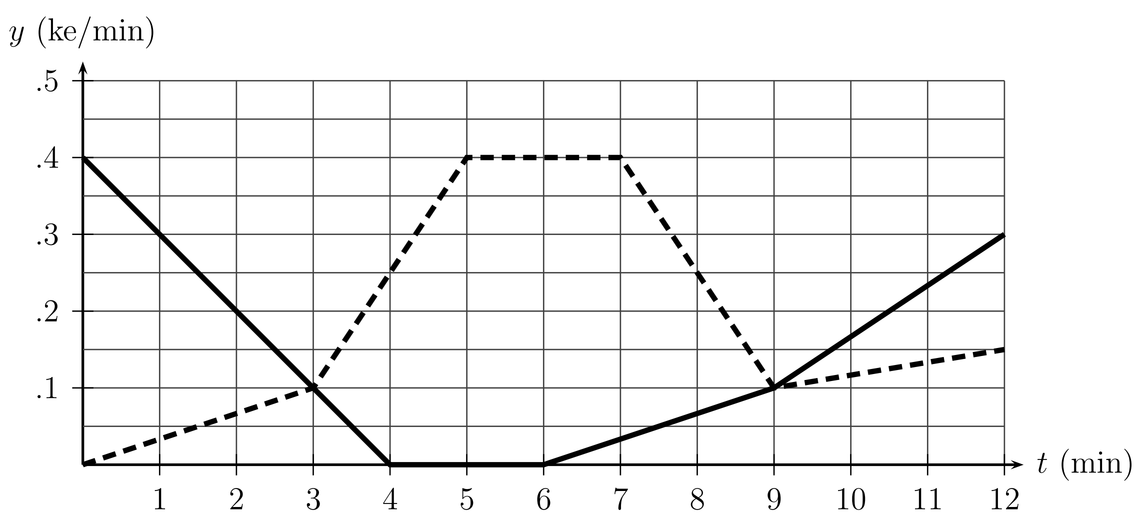
4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.



   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (Based on a Fall, 2011 Math 115 Exam Problem) Sally became an American citizen recently, so she celebrates by drinking bubble tea and watching ASMR videos. The videos generate a feeling of "low-grade euphoria," and the dashed line below represents $a(t)$, the rate at which good feelings are generated, in units of kiloendorphins per minute. Unfortunately, slurping bubble tea can make it hard to hear, so it cancels out the soothing sounds. The solid line represents $b(t)$, the negative effect of slurping sounds, in the same units. At time 0 Sally has 18 kiloendorphins of euphoria.



   (a) Over what period(s) was Sally becoming happier?

   (b) When was her happiness increasing the fastest? When was it decreasing the fastest?

   (c) When was her happiness the greatest? Explain.

   (d) How happy (in kiloendorphins) was Sally at time $t = 12$?

   (e) Draw a graph of Sally's happiness as a function of time. Label all critical points and inflection points.