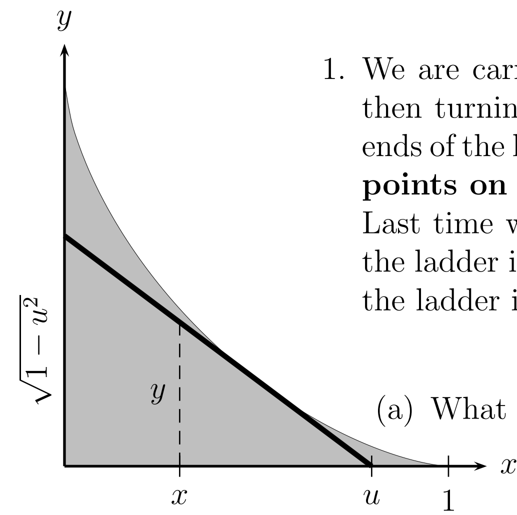
Douglass Houghton Workshop, Section 1, Wed 12/04/19

Worksheet Panda Bear



1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
   $$y = \frac{u-x}{u}\sqrt{1-u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)
   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?
   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

2. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:
   (a) $P'(2) = 55$
   (b) $\int_5^{10} P'(t)\,dt$
   (c) $P^{-1}(690)$
   (d) $\frac{1}{5}\int_5^{10} P(t)\,dt$

3. (This problem appeared on the Winter, 2015 Math 115 Final Exam) For nonzero constants a and b with $b > 0$, consider the family of functions given by
   $$f(x) = e^{ax} - bx.$$
   (a) Suppose the values of a and b are such that $f(x)$ has at least one critical point. For the domain $(-\infty, \infty)$, find all critical points of $f(x)$, all values of x at which $f(x)$ has a local extremum, and all values of x at which $f(x)$ has an inflection point. (Note that your answer(s) may include the constants a and/or b.)
   (b) Which of the following conditions on the constant a guarantee(s) that $f(x)$ has at least one critical point in its domain $(-\infty, \infty)$?
   (i) $a < 0$ (ii) $0 < a < b$ (iii) $b < a$
   (c) Find exact values of a and b so that $f(x)$ has a critical point at $(1, 0)$.

4. (Fall, 2013) Amanda is running sprints in Crisler Center. She begins in the middle of the "M" at the center of the court and runs north and south. Her velocity, in meters per second, for the first 9 seconds is $v(t) = t\sin(\frac{\pi}{3}t)$, where t is the number of seconds since she started running. She is running north when $v(t)$ is positive and south when $v(t)$ is negative.

   (a) Show that $f(t) = \dfrac{9}{\pi^2}\sin\left(\dfrac{\pi}{3}t\right) - \dfrac{3}{\pi}t\cos\left(\dfrac{\pi}{3}t\right)$ is an antiderivative of $v(t)$.

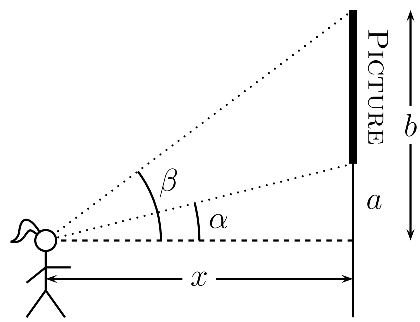
   (b) Where on the court is Amanda after 9 seconds?

   (c) What is the total distance traveled by Amanda in 9 seconds?

5. Suppose a picture is mounted on the wall. Its bottom is a feet above eye level, and its top is b feet above eye level. If you stand far away from the wall, you can't see the picture well. But if you stand close to the wall, you can't see well either! So the question is: how far from the wall should you stand in order to have the best view?

   (a) Let α and β be the angles between eye level and the bottom and top of the picture, as shown. x is your distance from the wall. Find α and β in terms of x, a, and b.

   (b) $\beta - \alpha$ is the angle that the picture takes up in your field of vision. So find the value of x that maximizes $\beta - \alpha$.



6. (Fall 2008) This problem was a smörgåsbord:

   (a) If $f(x)$ is even and $\int_{-2}^{2}(f(-x) - 3)\,dx = 8$, find $\int_0^2 f(x)\,dx$.

   (b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.

   (c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_3^{27} R(t)\,dt$ means in this context.

   (d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

   (e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \le 1 \\ 6\sin(\pi(x - 0.5)) + k & \text{if } x > 1 \end{cases}$$