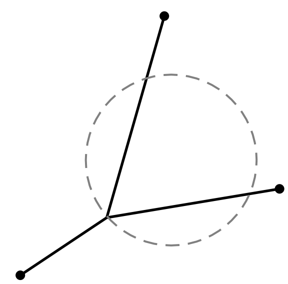
Douglass Houghton Workshop, Section 1, Mon 12/02/19

Worksheet Once More, Unto the Breach, Dear Friends

1. Shortest Network. Last time we used calculus to show that a Λ-shaped network can be improved if the vertex angle is less than 120°.

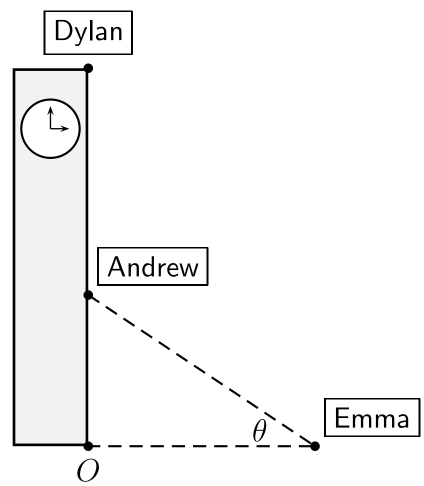
   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.

2. (Adapted from a Winter, 2005 Math 115 exam) One day Dylan notices that the door to the Burton Tower carillon has been left open. He can't resist the urge to climb to the top of the tower and barricade himself in, then serenade the campus with an extended saxophone solo.
   Andrew is working off some Chinese food he had over Thanksgiving break, so he begins to climb the outside of the tower to retake the carillon. Meanwhile on the ground, 30 feet from the tower, Emma improvises a ballet dance to go with the music. She looks up at an angle θ to see Andrew.

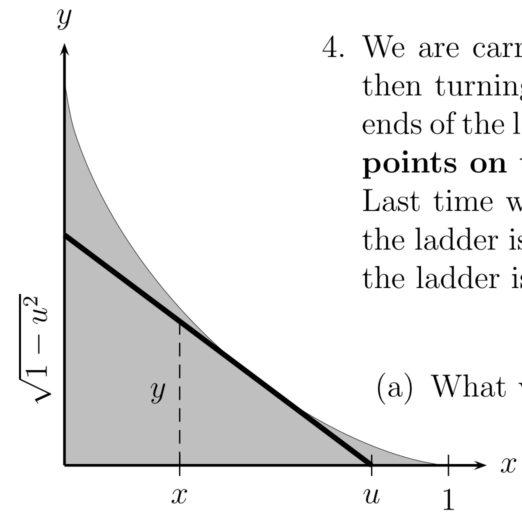
   (a) Find the rate of change of Andrew's distance from the point O with respect to θ.

   (b) If the distance from point O to Dylan is 200 ft and Andrew climbs at a constant 8 ft/sec, what is the rate of change of θ with respect to time when Andrew is halfway up?

   (c) When Andrew is halfway up, Dylan drops the end of a a rope down to help him. The end of the rope falls with a constant acceleration of 32 ft/sec^2. When does Andrew catch it, and what is its speed when he does?

   (d) Emma watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of her gaze changing when Andrew catches the rope?

3. (This problem appeared on a Winter, 2004 Math 115 Final Exam.) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$, measured in songs/hour where $t = 0$ corresponds to 5 pm. Explain the meaning of the quantity $\int_0^5 m(t)\,dt$.



4. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
$$y = \frac{u-x}{u}\sqrt{1-u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)
   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?
   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)

5. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

6. Suppose $\int_a^b f(x)\,dx = 2$ and $\int_a^b g(x)\,dx = 4$. Evaluate the following expressions, if possible. Assume that all functions are continuous on the interval $[a, b]$.
   (a) $\int_a^b (g(x))^2\,dx - \left(\int_a^b g(x)\,dx\right)^2$
   (b) $\int_{a+2}^{b+2} f(x-2)\,dx$
   (c) $\int_a^b (f(x)g(x))\,dx$
   (d) $\int_b^a (g(x))\,dx$

7. (This problem appeared on the Winter, 2004 Math 115 Final Exam) It is estimated that the rate people will visit a new theme park is given as
$$r(t) = \frac{A}{1 + Be^{-0.5t}}$$
   where A and B are both constants and $r(t)$ is measured in people per day, and $t = 0$ corresponds to opening day.
   (a) Write an integral that gives the total number of people visiting the park in the first year it is open. Do not try to evaluate the integral!
   (b) Suppose that $A = 100$ and $B = 5$. Given that
$$\frac{d}{dt}\left(2A\ln(1 + Be^{-0.5t}) - 2A\ln(Be^{-0.5t})\right) = \frac{A}{1 + Be^{-0.5t}},$$
   use the First Fundamental Theorem of Calculus to evaluate how many people visit the park during the first year it is open. Make sure you clearly indicate your use of the theorem.