

Worksheet Kangaroo

1. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing “Hail to the Victors” has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

- (a) How long should the band play to maximize the number of points Michigan scores?
(b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

- (c) What will be the score of the game for the case you found in part (b)?
2. In “The 12 days of Christmas”, a certain poultry-afficianado receives a number of gifts from her true love:

Day 1: A partridge in a pear tree. How to get it down?

Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

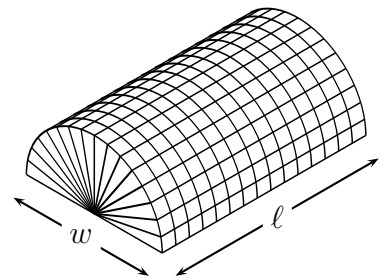
Day 3: 3 French hens, 2 more turtle doves, and another partidge.

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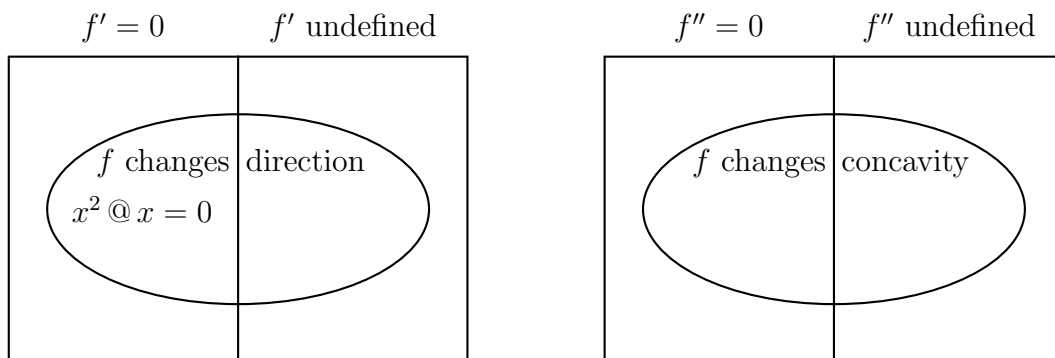
Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

- (a) If item 1 is “partridge”, item 2 is “turtle dove”, etc., then write a formula for the total number of item n ’s received.
(b) Of which item does Mr. Truelove send the most? (Solve using calculus.)
3. (This problem appeared on a Winter, 2008 Math 115 Exam.)
- (a) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f ?
(b) Find all critical points, local maxima, and local minima of f .
(c) Which of the local maxima and minima are global maxima / minima?

4. (From a Winter, 2011 Math 115 exam) A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ , is $1 + 0.001\ell$ dollars per square foot.



- (a) Write an equation for the cost of a hoophouse in terms of ℓ and w .
- (b) Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.
5. Christina drives east on the Ohio Turnpike to see her dogs, Wilbur and Tina, at home in New Jersey. She takes a ticket out of the machine in Toledo, and then turns it in at the toll booth near Columbus. Along with her change, the State Trooper in the toll booth hands Christina a speeding citation, and says that he *knows* Christina was going exactly 70 mph at some point on her trip. How does the Mean Value Theorem tell the trooper that?
6. The diagrams below each have 4 regions, representing different ways a function can behave at a point. In each region write an example of a function and a point that meets the criteria. For example, in the intersection of “ $f' = 0$ ” and “ f changes direction”, we have $x^2 @ x = 0$, because the derivative of x^2 is indeed 0 at $x = 0$, and the function switches from decreasing to increasing there.



7. Consider the function $f(x) = x^x$.
- (a) It's neither a power function (ax^b) nor an exponential (ab^x). Nevertheless, find its derivative. Hint: rewrite it in the form $e^{u(x)}$ for some function u .
- (b) What is the minimum value that f takes on? (Check with your calculator, but find the answer with calculus.)