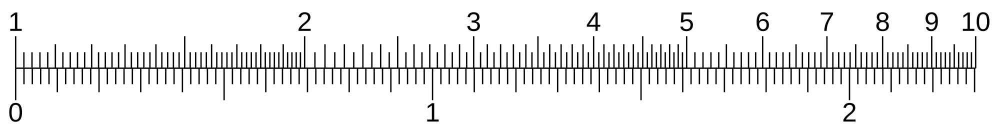
Douglass Houghton Workshop, Section 1, Wed 09/25/19

Worksheet Gobstopper

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

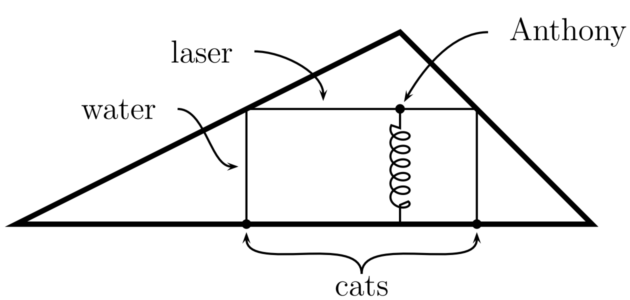
2. What does this picture represent?



3. Explain how to use two rulers to add numbers.

4. Explain how a slide rule is able to multiply two numbers.

5. Anthony is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Anthony goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Anthony's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Anthony's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Anthony is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Anthony's height off the floor.

6. Prove that it's possible to make a fair 5-sided die.

7. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

8. Last time we investigated rules for how a population of honeybees might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 200$	800	Yes

An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

$$P(n+1) = .4P(n) + 600 \qquad P(n+1) = -1.3P(n) + 460$$
$$P(n+1) = 1.1P(n) - 330 \qquad P(n+1) = P(n) + 300$$
$$P(n+1) = -.5P(n) + 1200 \qquad P(n+1) = -P(n) + 300$$

(b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

9. (This problem appeared on a Fall, 2015 Math 115 Exam) Cal is jumping a rope being swung by Gen and Algie while Maddy runs a stopwatch. There is a piece of tape around the middle of the rope. When the rope is at its lowest, the piece of tape is 2 inches above the ground, and when the rope is at its highest, the piece of tape is 68 inches above the ground. The rope makes two complete revolutions every second. When Maddy starts her stopwatch, the piece of tape is halfway between its highest and lowest points and moving downward. The height H (in inches above the ground) of the piece of tape can be modeled by a sinusoidal function $C(t)$, where t is the number of seconds displayed on Maddy's stopwatch.

(a) Sketch a well-labeled graph of two periods of $C(t)$ beginning at $t = 0$.

(b) Find a formula for $C(t)$.

(c) Now Gen takes a turn at jumping while Cal and Algie swing the rope. Maddy resets the stopwatch and starts it over again. Let $G(w)$ be the height (in inches above the ground) of the piece of tape when Maddy's stopwatch says w seconds. A formula for $G(w)$ is $G(w) = 41 + 38\cos(2\pi w)$. Maddy is 60 inches tall. For how long (in seconds) during each revolution of the rope is the piece of tape higher than the top of Maddy's head? (Assume Maddy is standing straight while watching the stopwatch.)