Worksheet Friends, Romans, and Countrymen

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

2. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

3. Ryan is studying a population of honeybees in Michigan. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

   where $P(0)$ is the population in 2019, $P(1)$ is the population 1 year later, etc. (P is measured in hundreds of bees.)

   (a) Make up a (short) story about honeybees that yields that formula as the result.

   (b) Suppose $P = 320$ in 2019. What will happen in the long run?

   (c) Suppose instead that $P = 800$ in 2019. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

4. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 125.$$

5. What's the deal with these pictures? What are they good for?
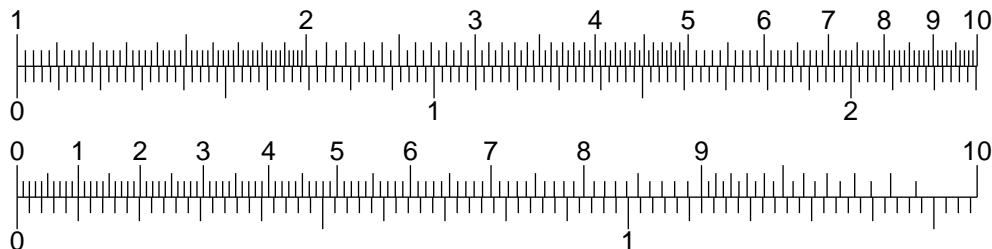
6. Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.

7. (This problem appeared on a Winter, 2012 Math 115 exam. Really!) Enjoying break-fast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

   (a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

   (b) According to your function, will the hull of the ship hit the reef? Explain.

   (c) What percentage of the time is the ship within 4 feet of the reef?

8. Jordan has noticed that her tastes changed over the last year. A year ago she spent about 15 hours a week cooking, and 10 hours playing soccer. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week cooking and 5 hours a week playing soccer.

   Let $C(t)$ be the number of hours Jordan spent cooking in week t, and let $S(t)$ be the number of hours she spent playing soccer. Assume $C(t)$ and $S(t)$ are continuous functions of time.

   (a) What does it mean for a function to be continuous?

   (b) Are $C(t) + S(t)$, $C(t) - S(t)$, and $C(t)S(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Jordan was spending the same amount of time cooking and playing soccer.

9. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

10. (This problem appeared on a Winter, 2014 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?