Douglass Houghton Workshop, Section 1, Wed 09/18/19

Worksheet Eat Cake (Let Them)

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

2. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

3. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head & Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

4. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{T}{n}\right)^n = e^T.$$

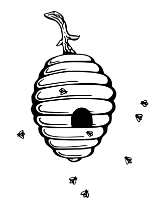
   (b) Devise a similar rule for the time it takes your money to triple at $r\%$ interest.

5. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

6. Ryan is studying a population of honeybees in Michigan. Suppose that the population changes according to the rule:



$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2019, $P(1)$ is the population 1 year later, etc. (P is measured in hundreds of bees.)

   (a) Make up a (short) story about honeybees that yields that formula as the result.

   (b) Suppose $P = 320$ in 2019. What will happen in the long run?

   (c) Suppose instead that $P = 800$ in 2019. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

7. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 125.$$

8. (This problem appeared on a Winter, 2009 Math 115 Exam) Air pressure, P, decreases exponentially with the height, h, in meters above sea level. The unit of air pressure is called an *atmosphere*; at sea level, the air pressure is 1 atm.

   (a) On top of Mount Denali, at a height of 6198 meters above sea level, the air pressure is approximately 0.48 atm. Use this to determine the air pressure 12 km above sea level, the maximum cruising altitude of a commercial jet.

   (b) Determine $P^{-1}(0.7)$. Include units!