Douglass Houghton Workshop, Section 1, Mon 09/16/19

Worksheet Dragon

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's apparently a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

   (a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

   (b) What's the formula for $N(T)$? (We found this previously).

   (c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

   (d) Verify your guess by finding a formula that fits the data.

   (e) Using the formula we found on Tuesday for splitting the towel into n parts, write a limit equation to express the result in part (d).

2. dBase$^{\text{TM}}$ was a database management system popular on IBM PCs back in the 80s, and still used in some places today. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

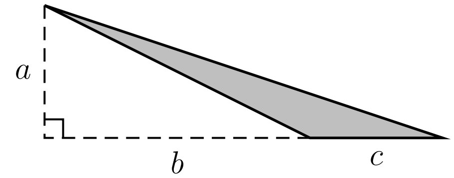
3. (This problem appeared on a Winter, 2017 Math 115 exam.) A company designs chambers whose interior temperature can be controlled. Their chambers come in two models: Model A and Model B.

   (a) The temperature in Model A goes from its minimum temperature of $-3\,^{\circ}$C to its maximum temperature of $15\,^{\circ}$C and returning to its minimum temperature three times each day. The temperature of this chamber at 10 am is $15\,^{\circ}$C. Let $A(t)$ be the temperature (in $^{\circ}$C) inside this chamber t hours after midnight. Find a formula for $A(t)$ assuming it is a sinusoidal function.

   (b) Let $B(t)$ be the temperature (in $^{\circ}$C) inside Model B t hours after midnight, where

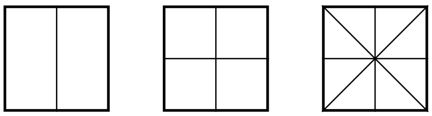
$$B(t) = 5 - 3\cos\left(\frac{3}{7}t + 1\right).$$

   Find the two smallest positive values of t at which the temperature in the chamber is $6\,^{\circ}$C. Your answer must be found algebraically. *Show all your work and give your answers in exact form.*



4. Find the area of the shaded triangle:

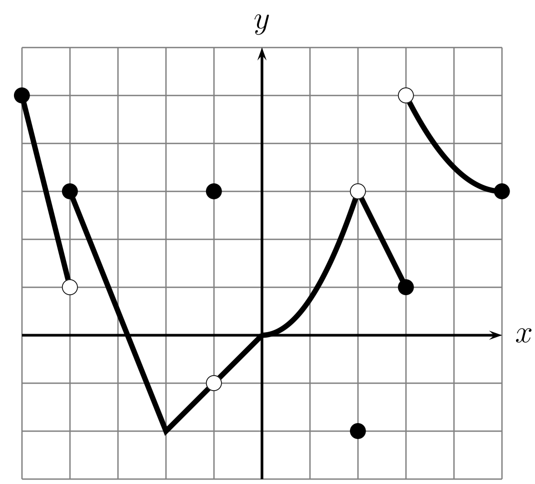
5. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a spice cake with creamy ginger frosting. It's getting a bit drippy while we decide how to cut it. Last week we figured out how to do it for $n = 2$, 4, and 8 people, and we had an idea for 16 people.



Can you prove that the idea we had for 16 people works? How about other numbers? Explain exactly how to cut the cake and why it works.

6. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

7. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $[-4, -1]$ $\quad [-2, 3]$ $\quad [2, 5]$ $\quad [-2, 2]$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim\limits_{x\to -1} Q(x)$ $\qquad$ ii. $\lim\limits_{w\to 2} Q(Q(w))$

   iii. $\lim\limits_{h\to 0} \dfrac{Q(-3+h) - Q(-3)}{h}$ $\qquad$ iv. $\lim\limits_{x\to\infty} Q\left(\dfrac{1}{x} + 3\right)$ $\qquad$ v. $\lim\limits_{x\to \frac{1}{3}} xQ(3x-5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim\limits_{x\to p^-} Q(x) \neq Q(p)$?

8. (This problem appeared on a Winter, 2013 Math 115 Exam.) The air in a factory is being filtered so that the quantity of a pollutant, P (in mg/liter), is decreasing exponentially. Suppose t is the time in hours since the factory began filtering the air. Also assume 20% of the pollutant is removed in the first five hours.

   (a) What percentage of the pollutant is left after 10 hours?

   (b) How long is it before the pollution is reduced by 50%?