Douglass Houghton Workshop, Section 1, Wed 09/11/19

Worksheet Canary

1. As we know, Samuel loves Rubik's puzzles. Besides the regular cube he has a $4 \times 4 \times 4$ cube, a 12-sided cube, a fifteen puzzle, and many more. Currently he has 40 puzzles. He's actually a little tired of Rubik's puzzles, but his well-meaning friends and family keep giving him new puzzles every year.

   Write formulas for the number of puzzles Samuel will have t years from now, under the following conditions:

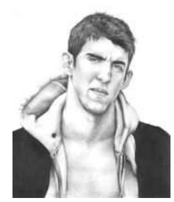
   (a) Samuel receives 5 new puzzles every year.

   (b) In year t Samuel receives one new puzzle for each two puzzles he had in year $t-1$.

   (c) Samuel receives 1 puzzle next year, 2 the year after that, 3 the year after that, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\mathrm{m}^2$, the towel is $T\,\mathrm{m}^2$, and he starts with 1 liter of water on him, we have

$$\text{wetness after regular toweling} = \frac{1}{1+T}$$
$$\text{wetness after “split” toweling} = \frac{1}{(1+T/2)^2}.$$



   Let's see just how much this "splitting" idea will buy us.

   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\mathrm{m}^2$					
$2\,\mathrm{m}^2$					
$4\,\mathrm{m}^2$					
$\frac{1}{2}\,\mathrm{m}^2$					

   (c) Consider the $1\,\mathrm{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. Write down the algebraic and geometric definitions of even and odd functions.

   (a) What kind of function do you get when you multiply two even functions? Write a proof, using the definitions.

   (b) How about the product of two odd functions?

   (c) Odd times even?

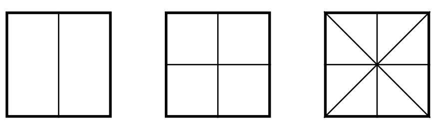
   (d) Odd plus odd, even plus even, odd plus even?

   (e) If a polynmial is odd, what can you say about it?

   (f) What if a polynomial is even?

   (g) A good crossword puzzle has 180-degree symmetry. Prove that if a is the number of across clues and d is the number of down clues, then the numbers a and d are either both even or both odd.

4. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a spice cake with creamy ginger frosting. It's getting a bit drippy while we decide how to cut it. Last week we figured out how to do it for $n = 2$, 4, and 8 people, and we had an idea for 16 people.

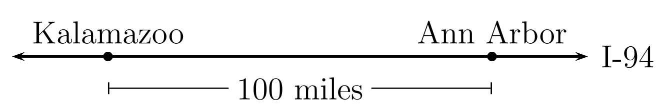


   Can you prove that the idea we had for 16 people works? How about other numbers? Explain exactly how to cut the cake and why it works.

5. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

6. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

   Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.