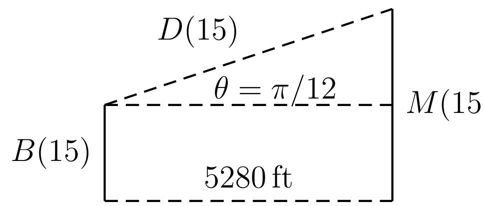
Douglass Houghton Workshop, Section 2, Tue 12/11/18

Worksheet To Infinity, and Beyond

1. (Adapted from a Fall, 2011 Math 115 Final Exam problem) Bri takes the train to New York City over winter break. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). Bri spots a Maserati sports car even with the train on the road, and turns her head as she watches it pull ahead. Let $M(t)$ be the distance between the car and its starting point, and $B(t)$ be Bri's distance from her starting point. After watching the car for 15 seconds, Bri has rotated her head $\pi/12$ radians.

   (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between Bri and the car 15 seconds after she starts watching it.



   (b) Let $\theta(t)$ be the angle Bri has turned her head after tracking the car for t seconds. Write an equation for the distance between Bri and the car at time t. (Your answer may involve $\theta(t)$.)

   (c) If at precisely 15 seconds, Bri is turning her head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between Bri and the car?

   (d) What is the speed of the car at 15 seconds?

2. David is running sprints in Crisler Center. He begins in the middle of the "M" at the center of the court and runs north and south. His velocity, in meters per second, for the first 9 seconds is $v(t) = t\sin(\frac{\pi}{3}t)$, where t is the number of seconds since he started running. He is running north when $v(t)$ is positive and south when $v(t)$ is negative.

   (a) Show that $f(t) = \dfrac{9}{\pi^2}\sin\left(\dfrac{\pi}{3}t\right) - \dfrac{3}{\pi}t\cos\left(\dfrac{\pi}{3}t\right)$ is an antiderivative of $v(t)$.

   (b) Where on the court is David after 9 seconds?

   (c) What is the total distance traveled by David in 9 seconds?

3. (Fall, 2014) Consider the family of functions given by

$$f(x) = \frac{ax}{e^{0.5(bx)^2}}$$

   where a and b are constants with $a > 1$ and $b > 1$. Find all the global extrema of f on the interval $\left[\frac{1}{4b}, \infty\right)$, and classify them as local maxes or local mins.

4. (This problem appeared on a Fall, 2005 Math 115 Final Exam) Using techniques from calculus, find the dimensions which will maximize the surface area of a circular cylinder whose height h and radius r, each in centimeters, are related by

$$h = 8 - \frac{r^2}{3}.$$

5. (Winter, 2011) A car speeds up at a constant rate from 10 to 70 mph over a period of half an hour, between $t = 0$ and $t = 1/2$. Its fuel efficiency, $E(v)$, measured in miles per gallon, depends on its speed, v, measured in miles per hour.

   (a) Write an integral for the total distance traveled by the car during the half hour.

   (b) Write an integral for the average fuel efficiency of the car during the half hour.

   (c) For speeds v greater than 70 mph suppose the relationship between E and v is given by

   $$E(v) = 2 + v^{-av}$$

   for some constant a. Using this formula, write an expression for the definition of the derivative $E'(82)$. Do not evaluate your expression.