Douglass Houghton Workshop, Section 2, Thu 12/07/17

Worksheet Some Rise By Sin, and Some By Virtue Fall

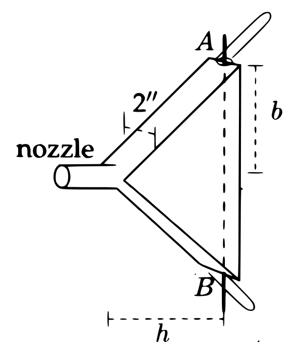
1. (Adapted from a Fall, 2004 Math 115 final) Bernie spends New Year's Day eating tamales. The rate at which he eats tamales is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in tamales/hour. Suppose $t = 0$ corresponds to 10 am.

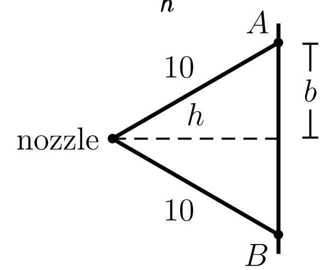
   (a) Write a definite integral that represents the total amount of tamales Bernie consumes between noon and 10 pm.

   (b) If Bernie's rate of eating tamales is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of tamales Bernie eats in the first four hours of his binge.

   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

2. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)





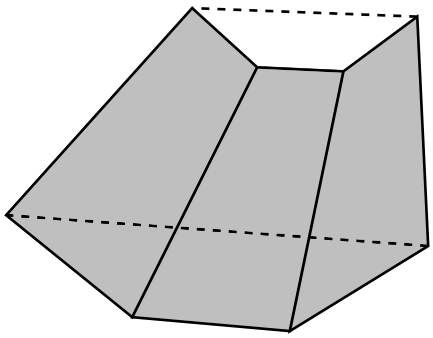
3. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:
$$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$

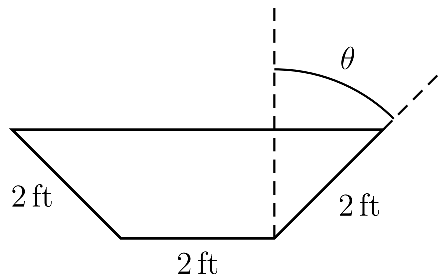
   (a) Find the zeroes of f in terms of A and B.

   (b) Find all critical points and inflection points of f in terms of A and B.

   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





   (a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

   (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (Fall, 2015) Consider the differentiable function Z defined by

$$Z(v) = \begin{cases} \dfrac{e^{v-1} - v}{(v-1)^2} & \text{if } v \neq 1 \\ \frac{1}{2} & \text{if } v = 1. \end{cases}$$

   Use the limit definition of the derivative to write an explicit expression for $Z'(1)$. *Your answer should not involve the letter Z.*

6. (Fall, 2016) Yukiko has a small orchard where she grows Michigan apples. After careful study last season, Yukiko found that the total cost, in dollars, of producing a bushels of apples can be modeled by

$$C(a) = -25500 + 26000e^{0.002a}$$

   for $0 \leq a \leq 320$. Qabil has promised to buy up to 100 bushels of apples for his famous apple ice cream. If Yukiko has any remaining apples, she has an agreement to sell them to Xanthippe's cider mill at a reduced price. Let $R(a)$ be the revenue generated from selling a bushels of apples. Then

$$R(a) = \begin{cases} 70a & \text{if } 0 \leq a \leq 100 \\ 2000 + 50a & \text{if } 100 < a \leq 320 \end{cases}$$

   (a) How much will Xanthippe's cider mill pay per bushel?

   (b) What is Yukiko's fixed cost?

   (c) For what values of a will Yukiko's marginal revenue equal her marginal cost?

   (d) Graph marginal revenue and marginal cost.

   (e) Assuming Yukiko can produce up to 320 bushels of apples, how many bushels should she produce, and what will be the maximum profit?