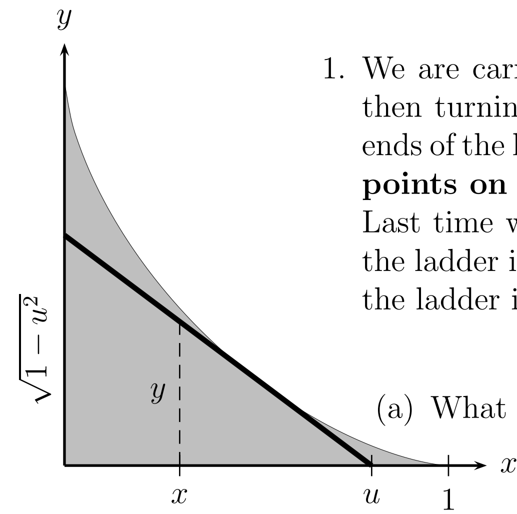
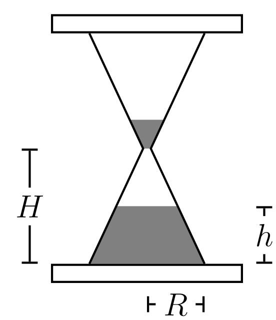
Douglass Houghton Workshop, Section 2, Tue 12/04/18

Worksheet Romeo, Romeo, Wherefore Art Thou Romeo?



1. We are carrying a ladder of length 1 down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?**
   Last time we found that if $0 \le x \le u \le 1$ and the base of the ladder is at $(u, 0)$, then the distance from $(x, 0)$ north to the ladder is
   $$y = \frac{u-x}{u}\sqrt{1-u^2}.$$
   (a) What value of u maximizes y? (Keep x fixed!)
   (b) So for a fixed x, what is the maximum value of y, as the ladder moves?
   (c) You have found a formula for the curve at the top of the region we want. Simplify until it's beautiful. (This is the best part, so don't stop until it's truly wondrous.)



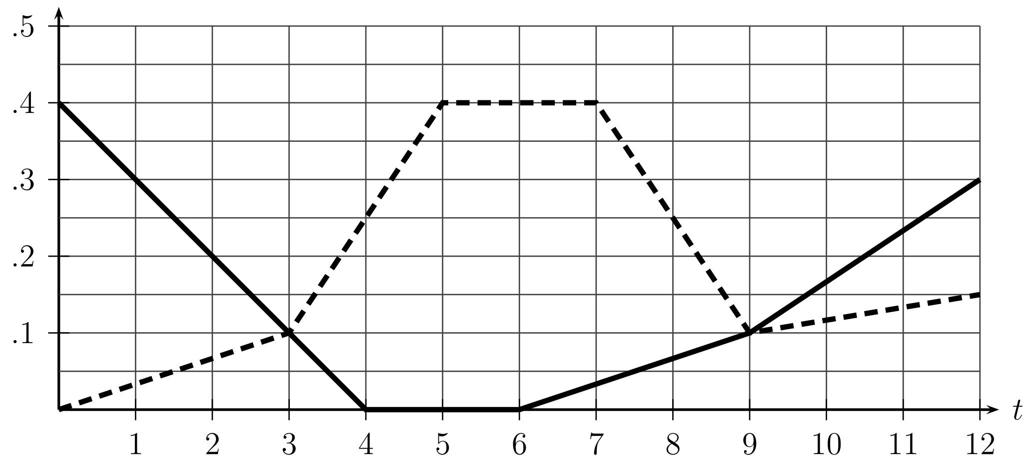
2. The lower chamber of an hourglass is shaped like a cone with height H inches and base radius R inches, as shown in the figure to the right, above. Sand falls into this cone. Write an expression for the volume of the sand in the lower chamber when the height of the sand there is h in (Hint: A cone with base radius r and height y has volume $V = \frac{1}{3}\pi r^2 y$, and it may be helpful to think of a difference between two conical vol umes.) Then, if $R = 0.9$ in, $H = 2.7$ in, and sand is falling into the lower chamber at $2\,\text{in}^3/\text{min}$, how fast is the height of the sand in the lower chamber changing when $h = 1$ in?

3. (Winter, 2010) Suppose that the standard price of a round-trip plane ticket from Detroit to Paris, purchased t days after April 30, is $P(t)$ dollars. Assume that P is an invertible function (even though this is not always the case in real life). In the context of this problem, give a practical interpretation for each of the following:

   (a) $P'(2) = 55$
   (b) $\int_5^{10} P'(t)\,dt$
   (c) $P^{-1}(690)$
   (d) $\frac{1}{5}\int_5^{10} P(t)\,dt$

4. (This problem appeared on a Fall, 2005 Math 115 Final Exam) Using techniques from calculus, find the dimensions which will maximize the surface area of a circular cylinder whose height h and radius r, each in centimeters, are related by
   $$h = 8 - \frac{r^2}{3}.$$

5. (Based on a Fall, 2011 Math 115 Exam Problem) Angel is working a 12-hour shift at the Detroit Public Library, re-shelving books in the basement. The rate $a(t)$ (in thousands of books per hour) at which she is adding books to the shelves is represented by the dashed line below in the graph. But as we know, the library is haunted, and at the same time Angel is putting books back on the shelves, a ghost is taking them off and leaving them on the floor. How annoying. The ghost's rate, $g(t)$, is represented by the solid line. At the beginning of the shift there were 18000 books on the shelves.



   (a) Over what period(s) was Angel working faster than the ghost?
   (b) When was the number of books on the shelves increasing the fastest? When was it decreasing the fastest?
   (c) When was the the number of books on the shelves the greatest? Explain.
   (d) How many books were on the shelves at the end of the shift (at $t = 12$)?
   (e) Draw a graph of the number of books on the shelves as a function of time. Label all critical points and inflection points.

6. (Fall 2008) This problem was a smörgåsbord:

   (a) If $f(x)$ is even and $\int_{-2}^{2}(f(-x) - 3)\,dx = 8$, find $\int_{0}^{2} f(x)\,dx$.
   (b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of c.
   (c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where t is measured in hours since noon on December 15, 2008), explain in words what $\int_{3}^{27} R(t)\,dt$ means in this context.
   (d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam t minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.
   (e) Find k so that the function $h(x)$ below is continuous for all x.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 1 \\ 6\sin(\pi(x - 0.5)) + k & \text{if } x > 1 \end{cases}$$