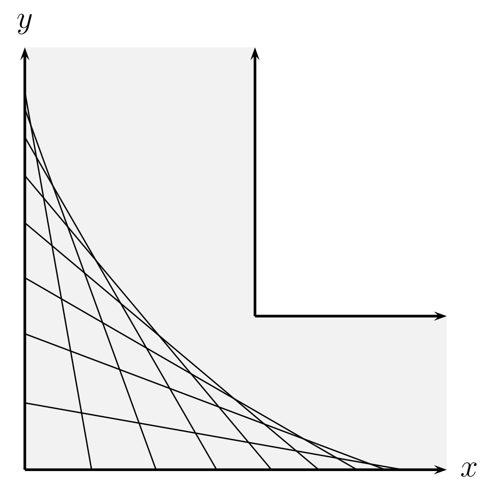
Douglass Houghton Workshop, Section 2, Thu 11/29/18

Worksheet Quoth the Raven, "Nevermore"

1. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

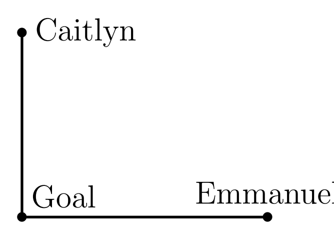
   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

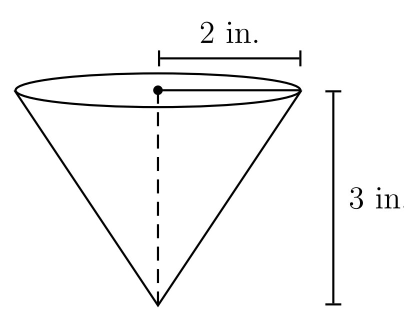
2. (Adapted from a Fall, 2005 Math 115 Final Exam) Emmanuel and Caitlyn are playing a combination of soccer and hockey on the edge of a frozen pond. Emmanuel kicks a soccer ball toward the goal, and Caitlyn skates toward the goal to catch it. The soccer ball's initial velocity is $30\,\text{ft/sec}$, but it decelerates $5\,\text{ft/sec}^2$ due to air resistance.

   (a) Emmanuel is 60 feet east of the goal, and Caitlyn is 30 feet north of the goal when Emmanuel kicks the soccer ball. What constant speed will Caitlyn have to skate in order to catch the soccer ball?

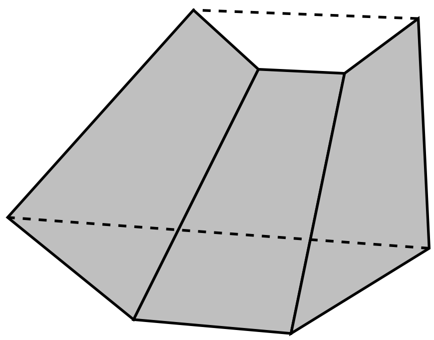
   (b) How fast is the distance between Caitlyn and the soccer ball changing when the soccer ball is halfway to the goal?

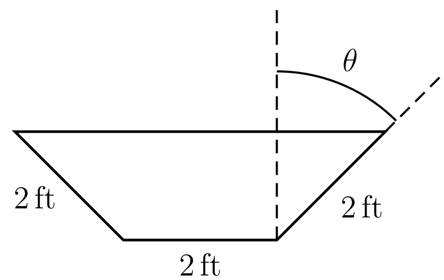


3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of 1.5 in^3/min when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.





(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (This problem appeared on the Winter, 2015 Math 115 Final Exam) For nonzero constants a and b with $b > 0$, consider the family of functions given by

$$f(x) = e^{ax} - bx.$$

(a) Suppose the values of a and b are such that $f(x)$ has at least one critical point. For the domain $(-\infty, \infty)$, find all critical points of $f(x)$, all values of x at which $f(x)$ has a local extremum, and all values of x at which $f(x)$ has an inflection point. (Note that your answer(s) may include the constants a and/or b.)

(b) Which of the following conditions on the constant a guarantee(s) that $f(x)$ has at least one critical point in its domain $(-\infty, \infty)$?

(i) $a < 0$ (ii) $0 < a < b$ (iii) $b < a$

(c) Find exact values of a and b so that $f(x)$ has a critical point at $(1, 0)$.

6. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.