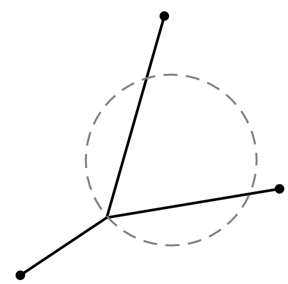
**Douglass Houghton Workshop, Section 2, Tue 11/27/18**

# Worksheet Past is Prologue

1. Shortest Network. Last time we used calculus to show that a $\Lambda$-shaped network can be improved if the vertex angle is less than 120°.

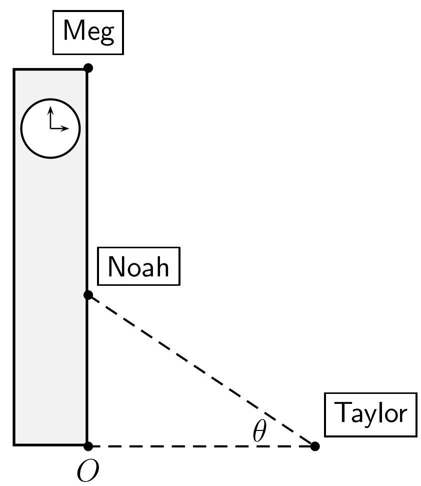
   (a) Prove that the network to the right is NOT minimizing. You don't need to find the optimal network, just prove that this one can be improved. Hint: consider the portion of the network that is inside the circle.

   (b) What allowed that trick to work? Phrase your answer like this: "Any network which contains ______ can be improved."

   (c) Put it all together, and explain where the soap puts the roundabout.

2. (Adapted from a Winter, 2005 Math 115 exam) One day Meg notices that the door to the Burton Tower carillon has been left open. She can't resist the urge to climb to the top of the tower and barricade herself in. She then begins a dramatic performance of selections from Miss Saigon, the musical she appeared in last year .
   Because he once climbed Mt. Ranier, The University asks Noah to scale the tower and retake the carillon. Meanwhile on the ground, 30 feet from the tower, Taylor is improvising an interpretive dance about the scene. She looks up at an angle $\theta$ to see Noah.

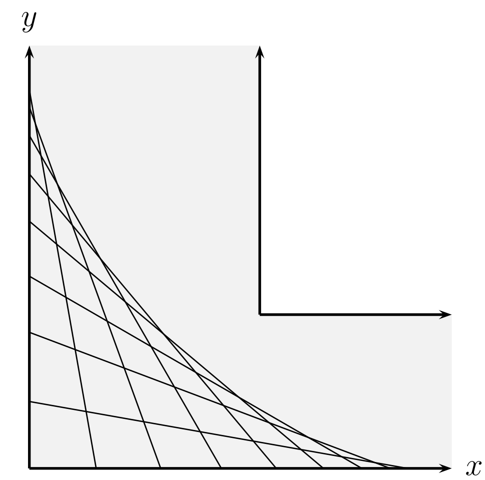
   (a) Find the rate of change of Noah's distance from the point $O$ with respect to $\theta$.

   (b) If the distance from point $O$ to Meg is 200 ft and Noah climbs at a constant 8 ft/sec, what is the rate of change of $\theta$ with respect to time when Noah is halfway up?

   (c) When Noah is halfway up, Meg drops the end of a rope down to help him. The end of the rope falls with a constant acceleration of 32 ft/sec$^2$. When does Noah catch it, and what is its speed when he does?

   (d) Taylor watches the end of the rope as it drops, and also begins backing away from the tower at a rate of 5 ft/sec. How fast is the angle of her gaze changing when Noah catches the rope?

3. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.




   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the $y$-axis is the top of the ladder? Draw a picture!

   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

4. Write the following sums in sigma ($\sum$) notation.

   (a) $1 + 2 + 3 + 4 + \cdots 10$

   (b) $1 + 2 + 3 + 4 + \cdots + n$

   (c) $3 + 5 + 7 + 9 + \cdots + 21$

   (d) $4 + 9 + 16 + 25 + \cdots + 100$

   (e) $2.3 + 2.8 + 3.3 + 3.8 + 4.3 + 4.8 + \cdots + 10.3$

   (f) $f(a_1) + f(a_2) + f(a_3) + \cdots + f(a_n)$

5. Suppose $\int_4^9 (4f(x) + 7)\,dx = 315$. Find $\int_4^9 f(x)\,dx$.

6. (This problem appeared on a Winter, 2004 Math 115 Final Exam.) As an avid online music trader, your rate of transfer of mp3's is given by $m(t)$, measured in songs/hour where $t = 0$ corresponds to 5 pm. Explain the meaning of the quantity $\int_0^5 m(t)\,dt$.

7. (Winter, 2004) Let $f$ be a continuous differentiable function of $x$. Suppose $f$ is always increasing. The following is a table of values of $f(x)$.

| $x$ | .8 | .9 | 1 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 |
|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 3 | 25 | 26 | 27 | 49 | 52 | 62 | 63 |

   (a) Using the table above, give an approximation of $f'(1)$.

   (b) Would a left-hand or a right-hand sum give a lower estimate of $\int_1^{1.5} f(x)\,dx$? Why?

   (c) Using the table above, give upper and lower estimates of $\int_1^{1.5} f(x)\,dx$.