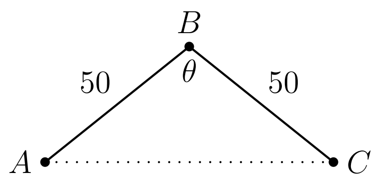
Douglass Houghton Workshop, Section 2, Tue 11/20/18

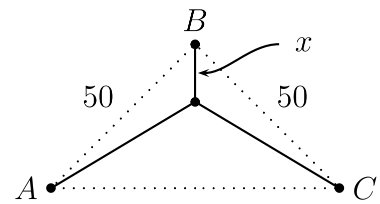
Worksheet Once More, Unto the Breach, Dear Friends

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.

(a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.



(b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

(c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

(d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

(e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. (Adapted from a Fall, 2005 Math 115 Final Exam) On the night of a particularly important high school football game, Erika and George are leading rival marching bands, converging on the stadium. They are trying to beat each other to midfield. Assume that Erika is due north of the stadium (therefore traveling due south) while George is directly east of the stadium, traveling due west. Erika's band is moving at 3 miles per hour, and George's at 2.8 miles per hour. How fast is the distance between them changing when Erika is 1.2 miles away and George is 1.6 miles away?

3. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Eve. At the end of every day this month she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

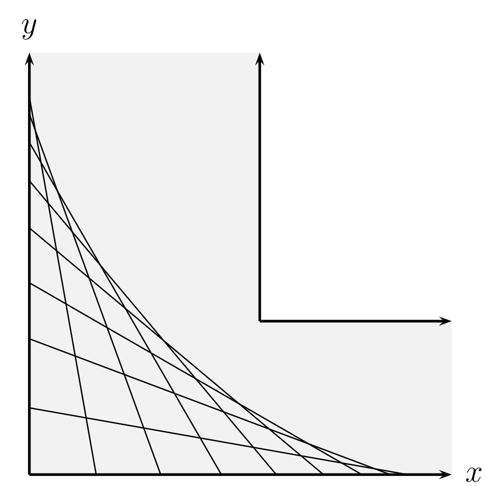
Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1800	5	8000	9	16300	13	21700	17	27300
2	3900	6	10700	10	17400	14	23100	18	30000
3	5100	7	12700	11	18700	15	24900	19	32500
4	6900	8	14600	12	20700	16	26200	20	32500

(a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Eve has written at time x. Assume that each day Eve writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

(b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

(c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.

4. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



(a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

(b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

To be continued...

5. (This problem appeared on a Winter, 2016 Math 115 FInal Exam.) Consider the family of functions given by $f(x) = e^{x^2+Ax+B}$ for constants A and B.

(a) Find and classify all local extrema of $f(x) = e^{x^2+Ax+B}$. Your answers may depend on A and/or B.

(b) Find the values of A and B that make $(3, 1)$ a critical point of $f(x)$.