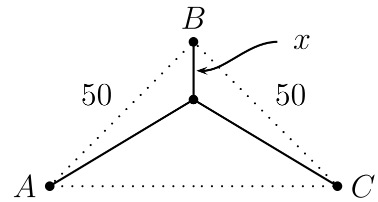
Douglass Houghton Workshop, Section 2, Thu 11/15/18

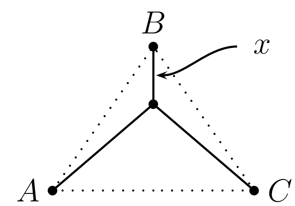
Worksheet Now is the Winter of our Discontent

1. We've been working on the problem of finding the shortest road network between three cities in the plane.

   In the case we considered, the three cities were at the corners of a 45°-45°-90° triangle with legs 50 miles long. The simplest idea is to just build roads along the legs; that makes a network of length 100. But by constructing a ⅄-shaped network like the one at the right, we found

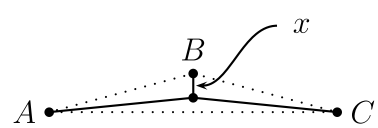
   

   - The length of the network is $x + 2\sqrt{2500 - 100x\cos(45) + x^2}$.
   - We can improve from the simple 2-road solution ($x = 0$, length $= 100$) by increasing x. For instance, when $x = 10$, the network has a length of about 97.

   (a) Consider the case where the triangle is still isosceles and the legs still have length 50, but the angle at B is 70°. Write a formula for the length of the network.

   (b) Can you find a value of x which beats the 2-road solution ($x = 0$, length $= 100$)?

   (c) Now suppose the vertex angle is very obtuse—say 150°. Find a formula for the length of the network.

   (d) Can you beat the 2-road solution in this case?

   (e) Suppose the vertex angle is θ. Write a formula for the length of the network.

2. (Adapted from a Fall, 2001 Math 115 final exam)

   Dalia and Nick start a business selling basball caps featuring both the logos of Michigan and their high school in Canton. Of course, they hope to encourage their high school friends to come to Michigan.

   Their cost and revenue functions are

   $$C(q) = 400 + 8q \qquad \text{and} \qquad R(q) = 60q^{.75}$$

   where q is the number of hats produced.

   (a) What is the product's fixed cost?

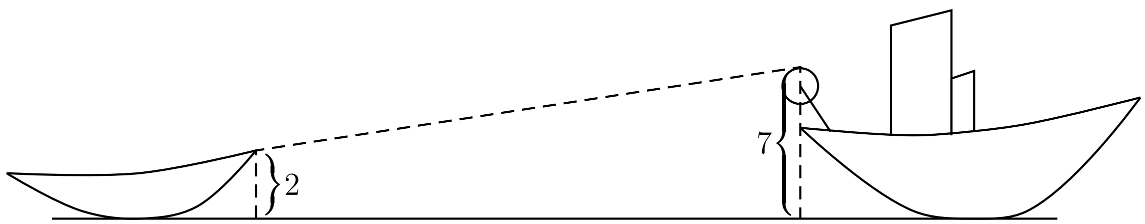
   (b) Last year, Dalia and Nick produced 2400 hats. What was their profit?

   (c) Find formulas for the marginal cost and marginal revenue, and evaluate at $q = 2400$.

   (d) Dalia and Nick would like to increase production and do better this year. Based on the marginal cost and marginal revenue *at this point* ($q = 2400$), explain whether their strategy is sound.

3. (Adapted from a Fall, 2006 Math 115 Final Exam.) Jessica is in the middle of a Dragon Boat race in New York Harbor, when the drummer, who keeps time for the rowers, suddenly breaks her drum in a fit of exuberance. Since a drummer is mandatory in dragon boat races, the team may not row home, and must hail a tugpoat to give them a tow.

   The tugboat captain throws Jessica a line, and she attaches it to her boat 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



   (a) At what rate is the length of the cable between the two boats changing?

   (b) How fast is Jessica's boat being pulled forward when it is 10 meters away from the tugboat?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day this month she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	4	7500	7	8700	10	9400	13	14800
2	3600	5	8300	8	8700	11	11800	14	17000
3	5800	6	8300	9	9400	12	13800		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.