Douglass Houghton Workshop, Section 2, Thu 11/01/18

Worksheet Magnificent

1. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.

(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

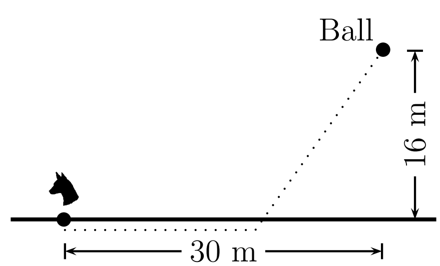
   (a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

   (b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

   (c) Can you find a value of x which will produce a shorter network than the simple proposal?

2. Suppose Emmanuel is walking along the shore of Lake Michigan with his dog Bella. Emmanuel throws a ball 30 meters down the beach and 16 meters out into the water.

Bella, being practical, wants to get to the ball as quickly as possible. The thing is that she can run faster than she can swim; her running speed on the beach is 9 meters per second, and she can swim 3 meters per second. How should Bella (who has an intuitive notion of calculus) get to the ball?



3. (This problem appeared on a Winter, 2004 Math 115 exam. Really!) While exploring an exotic spring break location, you discover a colony of geese who lay golden eggs. You bring 20 geese back with you. Suppose each goose can lay 294 golden eggs per year. You decide maybe 20 geese isn't enough, so you consider getting some more of these magical creatures. However, for each extra goose you bring home there are less resources for all the geese. Therefore, for each new goose the amount of eggs produced will decrease by 7 eggs per goose per year. How many more geese should you bring back if you want to maximize the number of golden eggs per year laid?

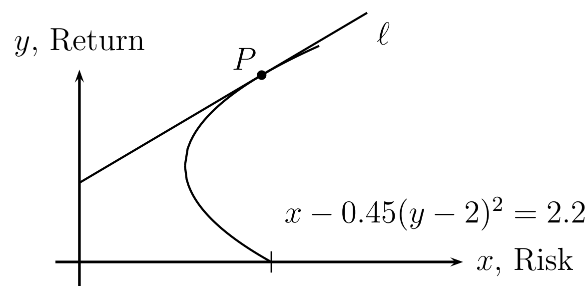
4. (This problem appeared on a Winter, 2008 Math 115 Exam.)

   (a) Consider the function $f(x) = x\sqrt{x+1}$. What is the domain of f?

   (b) Find all critical points, local maxima, and local minima of f.

   (c) Which of the local maxima and minima are global maxima / minima?

5. (This problem appeared on a Fall, 2008 Math 115 Exam) In Modern Portfolio Theory, a client's portfolio is structured in a way that balances risk and return. For a certain type of portfolio, the risk, x, and return, y, are related by the equation $x - 0.45(y-2)2 = 2.2$. This curve is shown in the graph below. The point P represents a particular portfolio of this type with a risk of 3.8 units. The tangent line, ℓ, through point P is also shown.

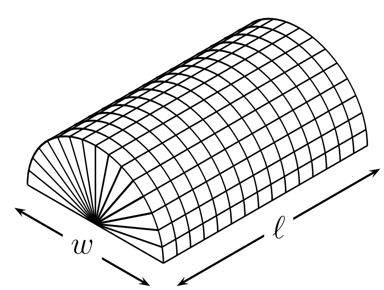


   (a) Using implicit differentiation, find dy/dx, and the coordinates of the point(s) where the slope is undefined.

   (b) The y-intercept of the tangent line for a given portfolio is called the *Risk Free Rate of Return*. Use your answer from (a) to find the Risk Free Rate of Return for this portfolio.

   (c) Now, estimate the return of an optimal portfolio having a risk of 4 units by using your information from part (b). Would this be an overestimate or an underestimate? Why?

6. (From a Winter, 2011 Math 115 exam) A hoophouse is an unheated greenhouse used to grow certain types of vegetables during the harsh Michigan winter. A typical hoophouse has a semi-cylindrical roof with a semi-circular wall on each end (see figure to the right). The growing area of the hoophouse is the rectangle of length ℓ and width w (each measured in feet) which is covered by the hoophouse. The cost of the semi-circular walls is \$0.50 per square foot and the cost of the roof, which varies with the side length ℓ, is $1 + 0.001\ell$ dollars per square foot.



   (a) Write an equation for the cost of a hoophouse in terms of ℓ and w.

   (b) Find the dimensions of the least expensive hoophouse with 8000 square feet of growing area.