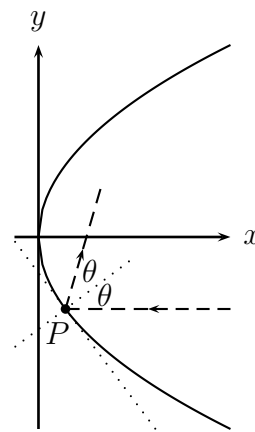


Worksheet Labradoodle

1. We're thinking about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$.

So far we've found:

- A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
- The slope of the tangent at that point is $-2/b$.
- The normal line at the same point has slope $b/2$.
- When a line makes an angle θ with the x -axis, it has slope $\tan \theta$.
- So if we call the angle between the normal line and the horizontal θ , then $\theta = \tan^{-1}(b/2)$.
- Which means the angle of the reflected ray to the horizontal is 2θ .
- Also, $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$,
- So the slope of the reflected ray is $\frac{4b}{4-b^2}$.



- (a) Write an equation for the reflected ray.
- (b) Where does the reflected ray intersect the x -axis? What is surprising about this answer?
- (c) Graph several rays, with their reflections.
- (d) What's cool about this type of mirror?
2. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

Day 1: A partridge in a pear tree. How to get it down?

Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

Day 3: 3 French hens, 2 more turtle doves, and another partidge.

...

Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

- (a) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n 's received.
- (b) Of which item does Mr. Truelove send the most? (Solve using calculus.)

3. Noah drives east on the Ohio Turnpike to an art show. He takes a ticket out of the machine in Toledo, and then turns it in at the toll booth near Columbus. Along with his change, the State Trooper in the toll booth hands Noah a speeding citation, and says that he *knows* Noah was going exactly 70 mph at some point on his trip. How does the Mean Value Theorem tell the trooper that?
4. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing “Hail to the Victors” has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

- (a) How long should the band play to maximize the number of points Michigan scores?
- (b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.

- (c) What will be the score of the game for the case you found in part (b)?
5. Let’s see if we can prove that the derivative of $\sin(x)$ is $\cos(x)$. Remember that last time we showed that

$$\lim_{\theta \rightarrow 0} \frac{\sin(\theta)}{\theta} = 1.$$

- (a) Show that $\lim_{\theta \rightarrow 0} \frac{1 - \cos(\theta)}{\theta} = 0$. Hint: Multiply the top and bottom by $1 + \cos(\theta)$, and simplify.
- (b) Write down the definition of the derivative of $\sin(x)$ at $x = a$.
- (c) Use a trig identity to write $\sin(a + h)$ in terms of sines and cosines of a and h .
- (d) Now use the two limits we know (the one from last time and the one in part (a)) to simplify the derivative.