

## Worksheet Incurrigible

1. While on a camping trip with his family, Max drops a rock off the edge of the Pictured Rocks to measure the height of the cliff.

- (a) The rock's downward acceleration due to gravity is  $-32 \text{ ft/sec}^2$ . So what is its velocity  $t$  seconds after it is dropped? (Keep in mind that the derivative of velocity is acceleration, and the initial velocity is 0.)
- (b) How far has the rock fallen after  $t$  seconds?
- (c) Suppose the rock takes 3.5 seconds to hit the bottom. How high are the cliffs?
- (d) Explain a general method for finding the height of any cliff.



2. Suppose you construct a  $1/z$  scale model of the White House, in order to film it blowing up. You will show the film at 24 frames per second. How many frames per second should you *film* so that when you slow the speed down, things will fall at believable speeds?



3. Suppose you are asked to design the first ascent and drop for a new roller coaster at Cedar Point. (You get to name it, too!) By studying photographs of your favorite coasters, you decide to make the slope of the ascent 0.8 and the slope of the drop  $-1.6$ . You decide to connect these two straight inclines  $y = L_1(x)$  and  $y = L_2(x)$  with part of a parabola  $y = f(x) = ax^2 + bx + c$ , where  $x$  and  $f(x)$  are measured in feet. For the track to be smooth there can't be abrupt changes in direction, so you want the linear segments  $L_1$  and  $L_2$  to be tangent to the parabola at the transition points  $P$  and  $Q$ . To simplify the equations, you decide to put the origin at  $P$ .

- (a) Name your coaster.

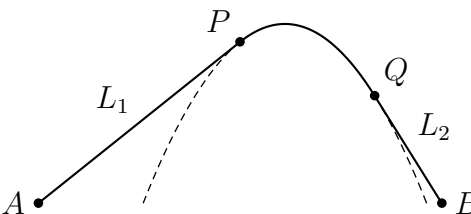
- (b) Suppose the horizontal distance between  $P$  and  $Q$  is 100 feet. Write equations in  $a$ ,  $b$ , and  $c$  that will ensure that the track is smooth at the transition points.

- (c) Solve the equations in (3b) for  $a$ ,  $b$ , and  $c$  to find a formula for  $f(x)$ .

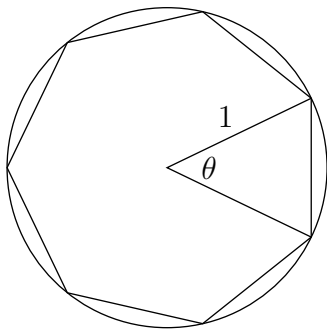
- (d) Plot  $L_1$ ,  $f$ , and  $L_2$  to verify graphically that the transitions are smooth.

- (e) Find the difference in elevation between  $P$  and  $Q$ .

- (f) Suppose the base of the hill (the distance from  $A$  to  $B$  in the picture) is 300 feet long. How high is the hill?



4. Find the quadratic polynomial that best approximates  $f(x) = 2^x$  at  $x = 1$ . (Hint: make sure that your quadratic and  $f$  have equal function values, equal first derivatives, and equal second derivatives at  $x = 1$ .) Then graph both on your calculator.
5. Let  $f(x) = x^2$  and  $g(x) = \frac{1}{3}x^3$ .
- Sketch the graphs of both functions on the same grid, for  $x \in [-1, 3]$ .
  - The vertical line  $x = x_0$  intersects the two graphs at  $(x_0, f(x_0))$  and  $(x_0, g(x_0))$ . For which vertical lines are the tangents at those points parallel? Try to guess the number of solutions by looking at the graph. Then calculate.
  - On which horizontal lines do the graphs have parallel tangents?
6. Let  $r$  be the radius of a sphere, and let  $V = f(r)$  be its volume.
- Find a formula for  $f(r)$ .
  - Find  $f'(r)$ . Do you recognize this formula? It's on the first page of your book.
  - Suppose a spherical balloon has radius 100 mm, and you pump it up so its radius is a small amount  $\Delta r$  bigger. What would happen to the volume? Answer in two ways:
    - By interpreting the derivative of volume, and
    - By imagining covering the balloon with tinfoil of thickness  $\Delta r$ .
  - Now explain the strange coincidence you found in (6b).
7. (This problem explains why  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ , but only when  $\theta$  is measured in radians.) Consider a regular  $n$ -sided polygon inscribed in a circle of radius 1.



- Let  $A_n$  be the area of the polygon. What does  $A_n$  approach as  $n$  gets large?  $\lim_{n \rightarrow \infty} A_n = \square$
  - We can compute  $A_n$  by dividing the polygon up into triangles which have a vertex at the center. Let  $\theta$  be the vertex angle (in radians). What is  $\theta$  in terms of  $n$ ?
  - What happens to  $\theta$  as  $n$  gets large?
  - What is the area of one of the triangles, in terms of  $\theta$ ?
- What is  $A_n$  in terms of  $\theta$ ?
  - Substitute into the equation from part (a) so that it includes  $\theta$ 's but not  $n$ 's. Simplify it as much as you can. Hint:  $\sin(2x) = 2 \sin(x) \cos(x)$ .
  - What would change if we measured  $\theta$  in degrees instead of radians?