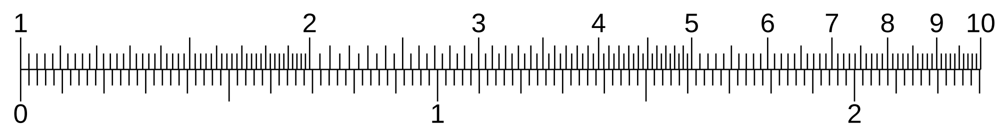
Douglass Houghton Workshop, Section 2, Tue 09/25/18

Worksheet Go Forth and Multiply

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

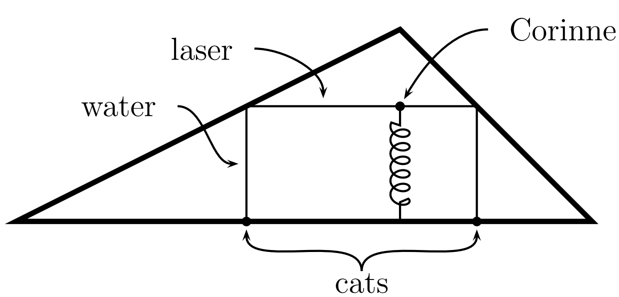
2. What does this picture represent?



3. Explain how to use two rulers to add numbers.

4. Explain how a slide rule is able to multiply two numbers.

5. Corinne is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from her head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Corinne goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Corinne's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Corinne's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Corinne is near the end of her journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Corinne's height off the floor.

6. Last time we tried making fair dice of various sizes. Let's nail down all the numbers you can make fair dice for. Rules:

   (a) All sides must be flat,

   (b) It must be equally likely to land on all sides, and

   (c) No handles (ala a dreidel).

   Hint: Do the previous problem first.

7. Last time we investigated rules for how a population of geese might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 200$	800	Yes

   An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

   (a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

   $$P(n+1) = .4P(n) + 600 \qquad P(n+1) = -1.3P(n) + 460$$
   $$P(n+1) = 1.1P(n) - 330 \qquad P(n+1) = P(n) + 300$$
   $$P(n+1) = -.5P(n) + 1200 \qquad P(n+1) = -P(n) + 300$$

   (b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

8. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.