Douglass Houghton Workshop, Section 2, Thu 09/20/18

Worksheet Fluffernutter

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

2. Eve is studying a population of geese on North Campus. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2018, $P(1)$ is the population 1 year later, etc.

   (a) Make up a (short) story about geese that yields that formula as the result.

   (b) Suppose there are 320 geese in 2018. What will happen in the long run?

   (c) Suppose instead that there are 800 geese in 2018. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

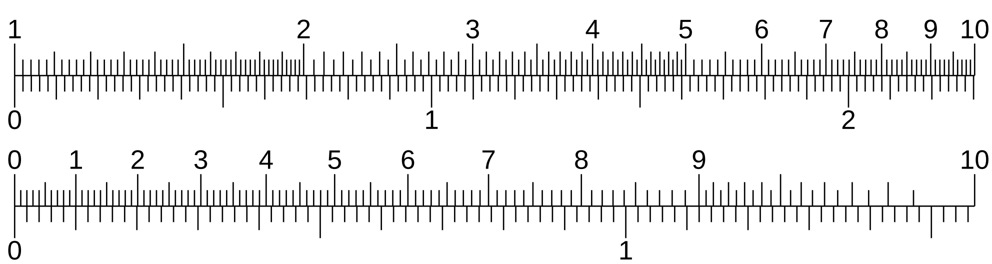
   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

3. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 200.$$

4. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two manatee scenarios has a stable equilibrium?

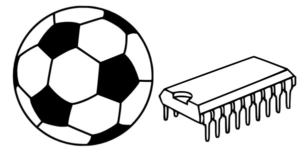
5. What's the deal with these pictures? What are they good for?



6. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

7. David has noticed that his tastes changed over the last year. A year ago he spent about 15 hours a week playing basketball, and 10 hours playing the saxophone. Gradually school took over his life, and though there have been some ups and downs in his schedule, the general trend is that he's spent less time per week on both. Now, 52 weeks later, he spends only 3 hours a week playing basketball and 5 hours a week playing the saxophone.

   Let $R(t)$ be the number of hours David spent playing basketball in week t, and let $M(t)$ be the number of hours he spent playing the saxophone. Assume $R(t)$ and $M(t)$ are continuous functions of time.

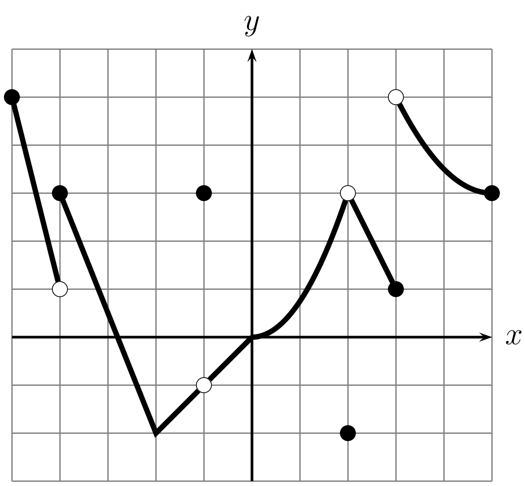


   (a) What does it mean for a function to be continuous?

   (b) Are $R(t) + M(t)$, $R(t) - M(t)$, and $R(t)M(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, David was spending the same amount of time playing basketball and playing the saxophone.

8. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



   (a) On which of the following intervals is $Q(x)$ invertible?

   $[-4, -1]$ $[-2, 3]$ $[2, 5]$ $[-2, 2]$

   (b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

   i. $\lim_{x \to -1} Q(x)$ ii. $\lim_{w \to 2} Q(Q(w))$

   iii. $\lim_{h \to 0} \frac{Q(-3+h) - Q(-3)}{h}$ iv. $\lim_{x \to \infty} Q\left(\frac{1}{x} + 3\right)$ v. $\lim_{x \to \frac{1}{3}} xQ(3x-5)$

   (c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

   (d) For which values of $-5 < p < 5$ is $\lim_{x \to p^-} Q(x) \neq Q(p)$?