Douglass Houghton Workshop, Section 2, Thu 09/13/18

Worksheet Down the Rabbit Hole

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's apparently a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

   Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

   (a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

   (b) What's the formula for $N(T)$? (We found this previously).

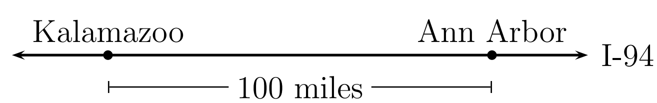
   (c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

   (d) Verify your guess by finding a formula that fits the data.

   (e) Using the formula we found on Tuesday for splitting the towel into n parts, write a limit equation to express the result in part (d).

2. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

   Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

3. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

4. dBase™ was a database management system popular on IBM PCs back in the 80s, and still used in some places today. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

5. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a yellow cake with chocolate frosting. It's getting a bit drippy while we decide how to cut it.

   We have a solution that seems to work, at least for 2, 4, 8, 16, and, we think, 20 people. So we should

   (a) Try to prove that our answer for 20 works,
   (b) Generalize to 12 and 24,
   (c) Generalize to all numbers,
   (d) Find a way to make each person's pieces adjacent, to reduce cutting as much as possible, and
   (e) Simplify the cuts needed as much as possible.

6. (This problem appeared on a Winter, 2016 Math 115 exam) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} xe^{Ax} + B & \text{if } x < 3 \\ C(x-3)^2 & \text{if } 3 \le x \le 5 \\ \frac{130}{x} & \text{if } x > 5 \end{cases}$$

   Suppose that $f(x)$ is continuous at $x = 3$, $\lim_{x\to 5^+} f(x) = 2 + \lim_{x\to 5^-} f(x)$, and $\lim_{x\to\infty} f(x) = -4$. Find A, B, and C.

7. Write down the algebraic and geometric definitions of even and odd functions.

   (a) What kind of function do you get when you multiply two even functions? Write a proof, using the definitions.
   (b) How about the product of two odd functions?
   (c) Odd times even?
   (d) Odd plus odd, even plus even, odd plus even?
   (e) If a polynmial is odd, what can you say about it?
   (f) What if a polynomial is even?
   (g) A good crossword puzzle has 180-degree symmetry. Prove that if a is the number of across clues and d is the number of down clues, then $a - d$ is an even number.