Douglass Houghton Workshop, Section 2, Tue 09/11/18

Worksheet Creamy Ginger Frosting

1. As we know, Jacob loves socks. He has three drawers; one for the graphical socks, one for the complex patterns, and one for the simple socks. Currently he has 40 pairs of socks. He's actually a little tired of socks, but his well-meaning friends and family keep giving him new pairs of socks every year.

   Write formulas for the number of pairs of socks Jacob will have t years from now, under the following conditions:

   (a) Jacob receives 5 new pairs of socks every year.

   (b) In year t Jacob receives one new pair of socks for each two pairs of socks he had in year $t-1$.

   (c) Jacob receives 1 pair of socks next year, 2 the year after that, 3 the year after that, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\mathrm{m}^2$, the towel is $T\,\mathrm{m}^2$, and he starts with 1 liter of water on him, we have

$$\text{wetness after regular toweling} = \frac{1}{1+T}$$
$$\text{wetness after “split” toweling} = \frac{1}{(1+T/2)^2}.$$

   Let's see just how much this "splitting" idea will buy us.

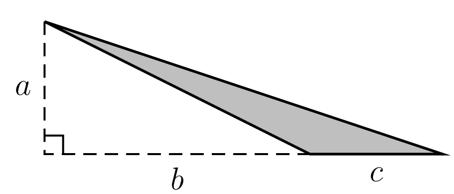
   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

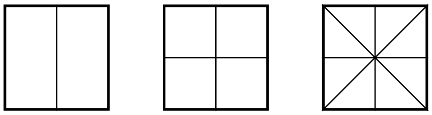
T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\mathrm{m}^2$					
$2\,\mathrm{m}^2$					
$4\,\mathrm{m}^2$					
$\frac{1}{2}\,\mathrm{m}^2$					

   (c) Consider the $1\,\mathrm{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. Find the area of the shaded triangle:



4. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a spice cake with creamy ginger frosting. It's getting a bit drippy while we decide how to cut it. Last week we figured out how to do it for $n = 2$, 4, and 8 people, and we had an idea for 16 people.

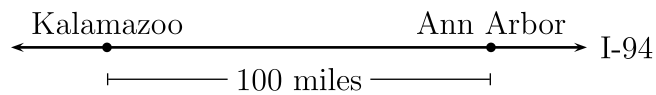


Can you prove that the idea we had for 16 people works? How about other numbers?

5. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

6. Kalamazoo is 100 miles west of Ann Arbor along Route 94.

   Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

7. (This problem appeared on a Fall, 2012 Math 115 exam) Suppose p represents the price of a reuben sandwich at a certain restaurant on State St. $R(p)$ represents the number of reubens the restaurant will sell in a day if they charge \$$p$ per reuben.

   (a) What does $R(5.5)$ represent in the context of this situation?

   (b) Assuming R is invertible, what does $R^{-1}(305)$ represent?

   (c) The owner of the restaurant also has a Church St. location. It doesn't get quite as much business, and the owner finds that the State St. store sells 35% more reubens than the Church St. store sells at the same price. Let $C(p)$ be the number of reubens the Church St location sells in a day at a price of \$$p$ each. Write a formula for $C(p)$ in terms of $R(p)$.

   (d) The owner starts doing research on reuben sales at the State Street location; he wants to know how the number of reubens sold is related to price. He finds that every time he raises the price by \$1 per reuben, the number sold in a day decreases by 20%. Let the constant B represent the number of reubens sold in a day at the State Street store if the price of reubens is \$5 each. Write a formula for $R(p)$ involving the constant B. Assume the domain of R is $1 \le p \le 25$.