Douglass Houghton Workshop, Section 1, Mon 12/10/18

Worksheet Question Everything

1. Adrian is studying a colony of *campylobacter jejuni* bacteria. He finds that the growth rate of the colony is increasing exponentially. That is, if $P(t)$ is the population in thousands after t hours, then $P'(t) = Ae^{kt}$ for some constants A and k.

   (a) Suppose there are 1000 bacteria at the start of the experiment. Write an integral which gives the number of bacteria present after T hours.

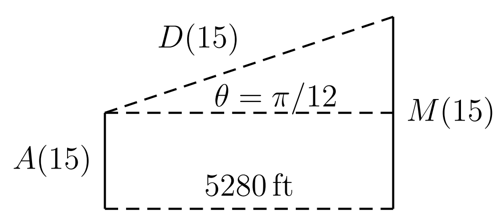
   (b) Use the Fundamental Theorem of Calculus to get a formula without an integral for the number of bacteria after T hours.

   (c) Suppose the bacteria grew at an initial rate of 500 bacteria per hour, and after 6 hours the rate has increased to 1000 bacteria per hour. Find values for the constants A and k.

   (d) How many bacteria are there 6 hours after the experiment started?

2. Hailey also does an experiment with the same starting population of bacteria, but she plays Coldplay to the bacteria. She finds they LOVE Coldplay, and they grow while a song is playing and stop growing between songs. She plays them a series of 3-minute songs with 3-minute breaks between them, and finds that t hours after the experiment starts, their growth rate (in thousands per hour) is $1 + \sin(20\pi t)$. How many bacteria grow in each 6-minute cycle?

3. (Adapted from a Fall, 2011 Math 115 Final Exam problem) David takes the train home to Chicago. At one point during the trip the tracks run parallel to a road, which is a mile away. The train is going quite slowly (6 ft/sec). David spots a Maserati sports car even with the train on the road, and turns his head as he watches it pull ahead. Let $M(t)$ be the distance between the car and its starting point, and $A(t)$ be David's distance from his starting point. After watching the car for 15 seconds, David has rotated his head $\pi/12$ radians.

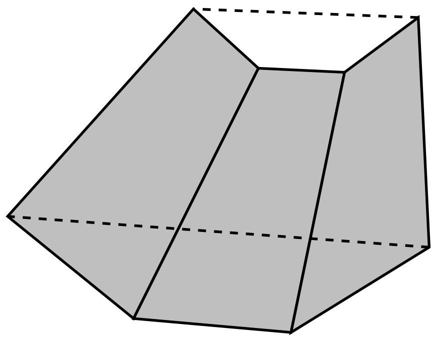
   (a) Initially the car is 1 mile (5280 ft) due east of the train. Find the distance between David and the car 15 seconds after he starts watching it.

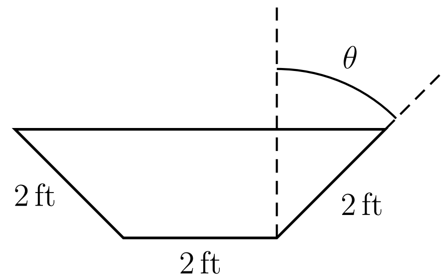
   (b) Let $\theta(t)$ be the angle David has turned his head after tracking the car for t seconds. Write an equation for the distance between David and the car at time t. (Your answer may involve $\theta(t)$.)

   (c) If at precisely 15 seconds, David is turning his head at a rate of .01 radians per second, what is the instantaneous rate of change of the distance between David and the car?

   (d) What is the speed of the car at 15 seconds?

4. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.

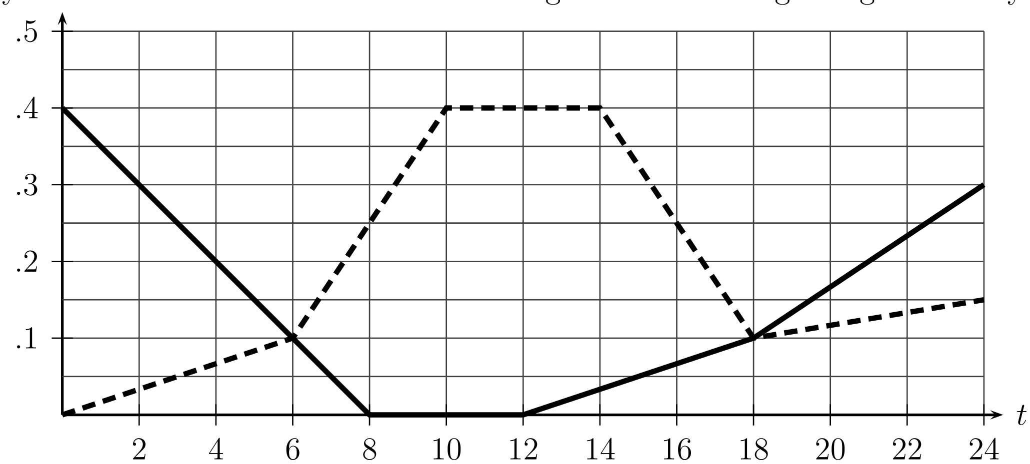




(a) What is the area, in terms of θ, of a cross section of the trough perpendicular to its long side? What is the volume of the trough?

(b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

5. (Fall, 2011) Suppose the graph below shows the rate of snow melt and snowfall on Mount Arvon, the highest peak in Michigan (at a towering 1970 ft), during a day (24 hour period) in April of last year. The function $m(t)$ (the solid curve) is the rate of snow melt, in inches per hour, t hours after the beginning of the day. The function $p(t)$ (the dashed curve) is the snowfall rate in inches per hour t hours after beginning of the day. There were 18 inches of snow on the ground at the beginning of the day.



(a) Over what period(s) was the snowfall rate greater than the snow melt rate?

(b) When was the amount of snow on Mount Arvon increasing the fastest? When was it decreasing the fastest?

(c) When was the amount of snow on Mount Arvon the greatest? Explain.

(d) How much snow was there on Mount Arvon at the end of the day (at $t = 24$)?

(e) Sketch a well-labeled graph of $P(t)$, an antiderivative of $p(t)$ satisfying $P(0) = 0$. Label and give the coordinates of the points on the graph of $P(t)$ at $t = 10$ and $t = 18$.