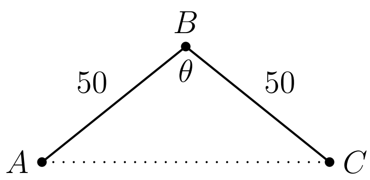
Douglass Houghton Workshop, Section 1, Wed 11/28/18

Worksheet Now is the Winter of our Discontent

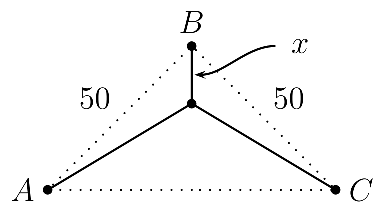
1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is $70°$ or $90°$, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is $150°$, the Λ is better than all possible ⅄'s.

   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the length of the ⅄-shaped network to the right, in terms of θ and x.

   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 50. Put the graphs on the board.

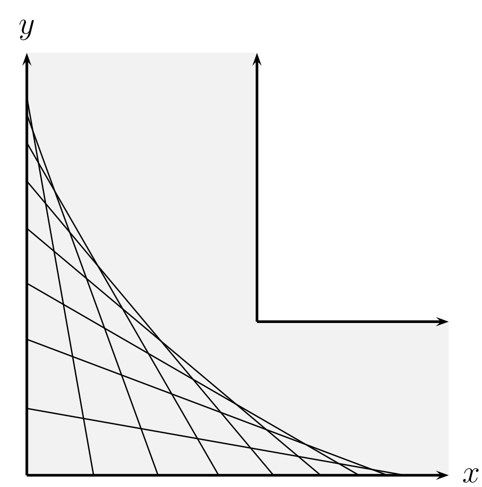


   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** Let's assume the ladder has length 1. In the picture, the gray area is the hallway and the fine lines are the ladder in different positions.



   (a) Suppose the base of the ladder is at the point $(u, 0)$. Where on the y-axis is the top of the ladder? Draw a picture!

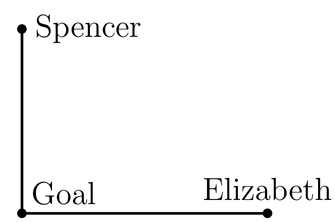
   (b) Suppose you are standing at $(x, 0)$ and looking north (up the page). If $x < u$, how far away do you see the ladder?

   To be continued...

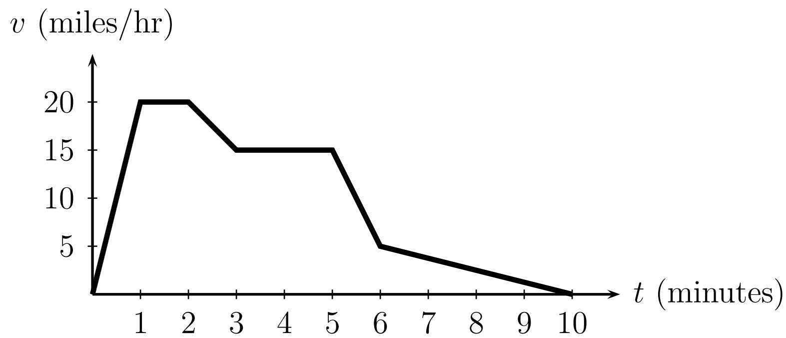
3. (Adapted from a Fall, 2005 Math 115 Final Exam) Elizabeth and Spencer are playing a combination of softball and tennis on the diag. Elizabeth throws a tennis ball toward the goal, and Spencer runs toward the goal to catch it. The tennis ball's initial velocity is 30 ft/sec, but it decelerates $5\,\text{ft/sec}^2$ due to air resistance.

   (a) Elizabeth is 60 feet east of the goal, and Spencer is 30 feet north of the goal when Elizabeth throws the tennis ball. What constant speed will Spencer have to run in order to catch the tennis ball?

   (b) How fast is the distance between Spencer and the tennis ball changing when the tennis ball is halfway to the goal?



4. (This problem appeared on the Winter, 2005 Math 115 Final Exam) In a certain episode of The Simpsons, Homer needs to deliver Lisa's homework to her at school, and he must do so before Principal Skinner arrives. Suppose Homer starts from the Simpson home in his car and travels with velocity given by the figure below. Suppose that Principal Skinner passes the Simpson home on his bicycle 2 minutes after Homer has left, following him to the school. Principal Skinner is able to sail through all the traffic and travels with constant velocity 10 miles per hour.



   (a) How far does Homer travel during the 10 minutes shown in the graph?

   (b) What is the average of Homers velocity during the 10 minute drive?

   (c) At what time, $t > 0$, is Homer the greatest distance ahead of Principal Skinner?

   (d) Does Principal Skinner overtake Homer, and if so, when? Explain.

   (e) If Skinner continues at 10 mph, and if Homer's velocity continues to decrease at a constant rate, then how long does it take for Homer and Skinner to meet?

5. (From the Winter, 2007 Math 115 Final Exam) Suppose that f and g are continuous functions with

$$\int_0^2 f(x)\,dx = 5 \qquad \text{and} \qquad \int_0^2 g(x)\,dx = 13.$$

   Compute the following. If the computation cannot be made because something is missing, explain clearly what is missing.

   (a) $\int_4^6 f(x-4)\,dx$

   (b) $\int_{-2}^0 2g(-t)\,dt$

   (c) $\int_2^0 (f(y)+2)\,dy$

   (d) $\int_2^2 g(x)\,dx$

   (e) Suppose that f is an even function. Find the average value of f from -2 to 2.