Douglass Houghton Workshop, Section 1, Mon 11/19/18

Worksheet Love All, Trust a Few, Do Wrong to None

1. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.

(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

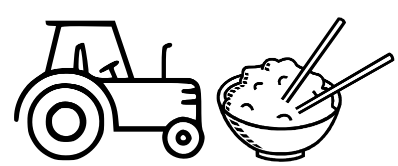
   (a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

   (b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

   (c) Can you find a value of x which will produce a shorter network than the simple proposal?

2. (Adapted from a Fall, 2001 Math 115 final exam)

   As we know, Calvin's home town hosts an annual Tractor Day. Calvin starts a business buying frozen Panda Express meals and reheating them for the tractor drivers. Naturally he delivers the food by tractor.



   His cost and revenue functions are

$$C(q) = 400 + 8q \qquad \text{and} \qquad R(q) = 60q^{.75}$$

   where q is the number of meals produced.

   (a) What is the product's fixed cost?

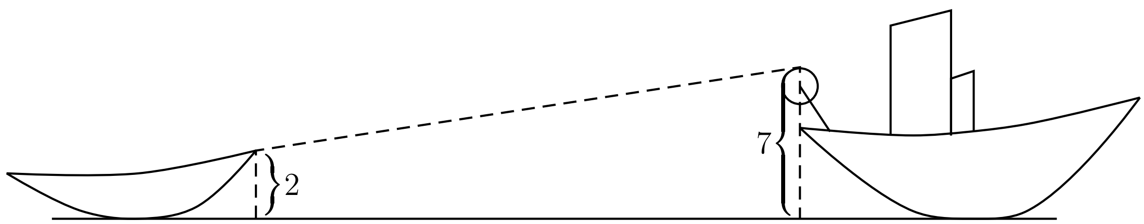
   (b) Last year, Calvin produced 2400 meals. What was his profit?

   (c) Find formulas for the marginal cost and marginal revenue, and evaluate at $q = 2400$.

   (d) Calvin would like to increase production and do better this year. Based on the marginal cost and marginal revenue *at this point* ($q = 2400$), explain whether his strategy is sound.

3. (Adapted from a Fall, 2006 Math 115 Final Exam.) Andrew is sailing a boat around his home of Grosse Ile, when the boat springs a leak. Being a Boy Scout he is, of course, prepared, so he patches the leak with duct tape. But in the process the sun goes down, so he reluctantly hails a passing tugboat to ask for a tow.

   The tugboat captain throws Andrew a line, and he attaches it to his boat 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



   (a) At what rate is the length of the cable between the two boats changing?

   (b) How fast is Andrew's boat being pulled forward when it is 10 meters away from the tugboat?

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day this month she uploaded her manuscript to a website (`nanowrimo.org`), which counts how many words she has written. Here are her counts, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	5	8300	9	9400	13	14800	17	23300
2	3600	6	8300	10	9400	14	17000	18	24100
3	5800	7	8700	11	11800	15	20100		
4	7500	8	8700	12	13800	16	21300		

   (a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for the second week (x from 7 to 14).

   (b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 7 to 14.

   (c) Now consider the function $F(t)$, which is the area between the line $x = 7$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(7), F(8), \ldots, F(14)$. What do you notice? Explain this result.