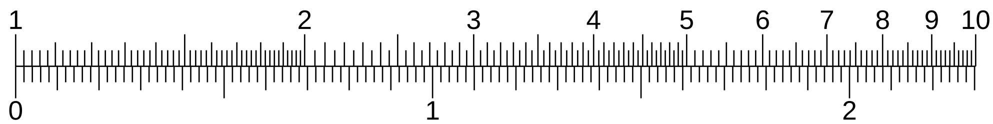
Douglass Houghton Workshop, Section 1, Wed 09/26/18

Worksheet Grand Inquisition

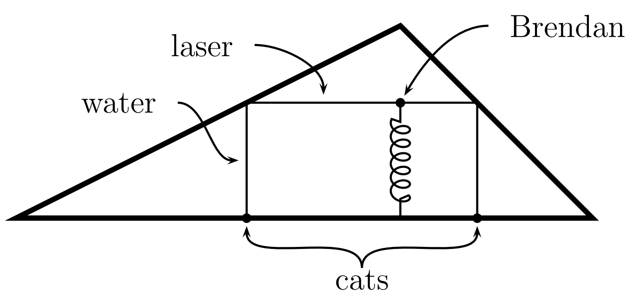
1. What does this picture represent?



2. Explain how to use two rulers to add numbers.

3. Explain how a slide rule is able to multiply two numbers.

4. Brendan is being raised on a giant spring-loaded platform right under the peak of a triangular room. Laser beams are being emitted from his head, parallel to the ground, until they hit the walls. Where they hit the walls, drops of water fall down, then land in the mouths of two cats. As Brendan goes up, the cats follow the drops toward the base of the platform.



   (a) Let $h(t)$ be Brendan's height at time t, and let $w(t)$ be the distance between the two cats. Are they continuous functions? Is $h(t) - w(t)$ a continuous function?

   (b) When t is close to 0 (so Brendan's head has just come through the floor), what can you say about $h(t) - w(t)$?

   (c) Later on, when Brendan is near the end of his journey and about to hit the top, what can you say about $h(t) - w(t)$?

   (d) Use the Intermediate Value Theorem to show that at some time the distance between the cats is the same as Brendan's height off the floor.

5. Last time we tried making fair dice of various sizes. Let's nail down all the numbers you can make fair dice for. Rules:

   (a) All sides must be flat,

   (b) It must be equally likely to land on all sides, and

   (c) No handles (ala a dreidel).

6. The *power rule for derivatives* says that if $f(x) = x^n$, then $f'(x) = nx^{n-1}$. Use the definition of the derivative to prove it for the case where n is a positive integer. Hint: Pascal's triangle.

7. Last time we investigated rules for how a population of polar bears might change. Let's nail down the essential features of all similar rules. Here's what we know:

Rule	Equilibrium	Stable?
$P(n+1) = 1.5P(n) - 200$	400	No
$P(n+1) = .75P(n) + 200$	800	Yes

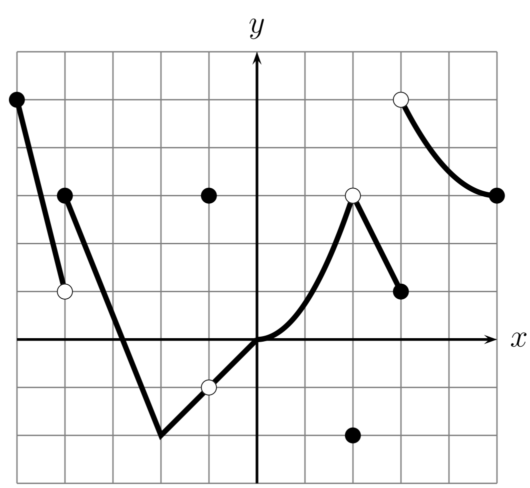
An **equilibrium** is a population that will stay constant from year to year. An equilibrium $\hat{P}$ is **stable** if when the population starts a little above or below $\hat{P}$, it moves toward $\hat{P}$. Otherwise $\hat{P}$ is **unstable**.

(a) Add rows to the table for these rules. You can reason either numerically, graphically, algebraically, or with words.

$P(n+1) = .4P(n) + 600$ $\qquad$ $P(n+1) = -1.3P(n) + 460$
$P(n+1) = 1.1P(n) - 330$ $\qquad$ $P(n+1) = P(n) + 300$
$P(n+1) = -.5P(n) + 1200$ $\qquad$ $P(n+1) = -P(n) + 300$

(b) Now do $P(n+1) = mP(n) + b$, where m and b are constants.

8. (From a Fall, 2017 Math 115 Exam.) The graph of $y = Q(x)$ is shown. The gridlines are one unit apart.



(a) On which of the following intervals is $Q(x)$ invertible?

$[-4, -1]$ $\quad$ $[-2, 3]$ $\quad$ $[2, 5]$ $\quad$ $[-2, 2]$

(b) Find the following limits. Write "NI" if there you don't have enough information and "DNE" if the limit doesn't exist.

i. $\lim_{x \to -1} Q(x)$ $\qquad$ ii. $\lim_{w \to 2} Q(Q(w))$

iii. $\lim_{h \to 0} \dfrac{Q(-3+h) - Q(-3)}{h}$ $\qquad$ iv. $\lim_{x \to \infty} Q\left(\dfrac{1}{x} + 3\right)$ $\qquad$ v. $\lim_{x \to \frac{1}{3}} xQ(3x-5)$

(c) For which values of $-5 < x < 5$ is the function $Q(x)$ <u>not</u> continuous?

(d) For which values of $-5 < p < 5$ is $\lim_{x \to p^-} Q(x) \neq Q(p)$?