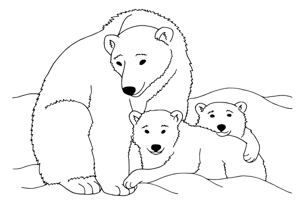
Douglass Houghton Workshop, Section 1, Mon 09/24/18

Worksheet Friends, Romans, and Countrymen

1. We're still sitting in this cabin, trying to figure out the temperature in Fahrenheit. We have a $33\frac{1}{3}$ RPM record player, a roll of duct tape, a Beatles album from 1967, a stopwatch, and some very cold feet. We know that if c is the temperature we read on the Celcius thermometer, then $f = \frac{9}{5}c + 32$ is the temperature in Fahrenheit. We need to convert from Celcius to Fahrenheit without multiplying or dividing.

2. Why is it necessary to define the derivative in terms of a limit? Draw a picture that describes how the derivative is the limit of the slopes of some lines.

3. Aniqa is studying a population of polar bears in Greenland. Suppose that the population changes according to the rule:



$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2018, $P(1)$ is the population 1 year later, etc.

   (a) Make up a (short) story about polar bears that yields that formula as the result.

   (b) Suppose there are 320 polar bears in 2018. What will happen in the long run?

   (c) Suppose instead that there are 800 polar bears in 2018. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

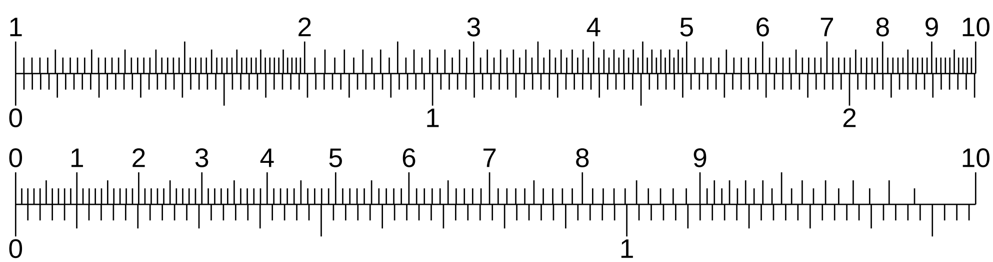
   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

4. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 200.$$

5. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two manatee scenarios has a stable equilibrium?

6. What's the deal with these pictures? What are they good for?



7. Use the definition of the derivative to find $f'(x)$ when $f(x) = \sqrt{x}$. Hint: $(a-b)(a+b) = a^2 - b^2$.

8. (This problem appeared on a Winter, 2012 Math 115 exam. Really!) Enjoying breakfast outdoors in a coastal Mediterranean town, Tommy notices a ship that is anchored offshore. The ship is stationed above a reef which lies below the surface of the water, and a series of waves causes its height to oscillate sinusoidally with a period of 6 seconds. When Tommy begins observing, the hull of the ship is at its highest point, 20 feet above the reef. After 1.5 seconds, the hull is 11 feet above the reef.

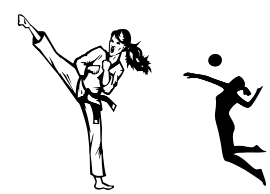
   (a) Write a function $h(t)$ that gives the height of the ship's hull above the reef t seconds after Tommy begins observing.

   (b) According to your function, will the hull of the ship hit the reef? Explain.

   (c) What percentage of the time is the ship within 4 feet of the reef?

9. Jacinda has noticed that her tastes changed over the last year. A year ago she spent about 15 hours a week practicing Tae Kwon Do, and 10 hours playing volleyball. Gradually school took over her life, and though there have been some ups and downs in her schedule, the general trend is that she's spent less time per week on both. Now, 52 weeks later, she spends only 3 hours a week practicing Tae Kwon Do and 5 hours a week playing volleyball.

   Let $T(t)$ be the number of hours Jacinda spent practicing Tae Kwon Do in week t, and let $V(t)$ be the number of hours she spent playing volleyball. Assume $T(t)$ and $V(t)$ are continuous functions of time.

   (a) What does it mean for a function to be continuous?

   (b) Are $T(t) + V(t)$, $T(t) - V(t)$, and $T(t)V(t)$ continuous?

   (c) Use one of the functions in part (a) together with the Intermediate Value Theorem to show that at some time in the last year, Jacinda was spending the same amount of time practicing Tae Kwon Do and playing volleyball.

10. Last time we tried making fair dice of various sizes. Let's nail down all the numbers you can make fair dice for. Rules:

    (a) All sides must be flat,

    (b) It must be equally likely to land on all sides, and

    (c) No handles (ala a dreidel).