Douglass Houghton Workshop, Section 1, Mon 09/11/17

Worksheet Beatles

1. Is it possible to dig a square hole in a triangular plot of land, in such a way that all four corners of the square are on the sides of the triangle?

2. Last time we found that when Michael Phelps dries himself off with a towel the same size as he is, then he ends up half as wet as he was before. And if the towel is twice as big as he is, he's a third as wet. But then we realized that if he just divides the big towel in half, he can get even drier.

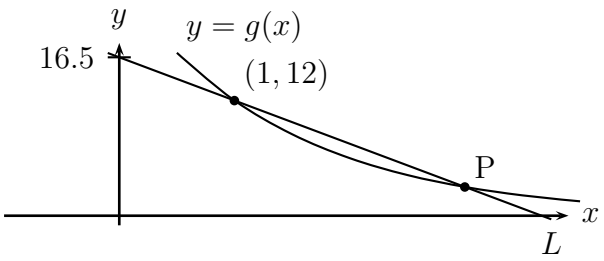
   (a) Assume again that Michael's surface area is $1\,\mathrm{m}^2$, and he starts with 1 liter of water on him. Fill in the table below with his wetness after drying in the normal way, and after drying by splitting the towel into two pieces:

Towel size	Wetness after "normal" towelling	Wetness after "split" toweling
$1\,\mathrm{m}^2$	$1/2\ \ell$	
$2\,\mathrm{m}^2$	$1/3\ \ell$	$1/4\ \ell$
$4\,\mathrm{m}^2$		
$8\,\mathrm{m}^2$		

   (b) What if the towel has surface area T? Find formulas for the two columns in terms of T.

   (c) Can you get him even drier with the same towel?

3. Consider the following graph:

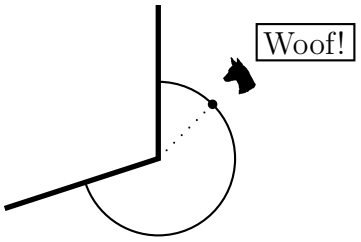


   (a) Find an equation for the line L.

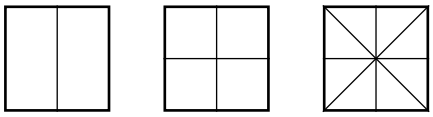
   (b) Find the y-coordinate of the point P, given that its x-coordinate is 3.

   (c) $g(x)$ is an exponential function. Find a formula for $g(x)$.

4. The security detail at the Pentagon in Arlington, VA, has chained five Rottweilers to each of the five corners of the building. Each chain is 100 feet long. Find the length of the path a single Rottweiler can walk, from one wall to the other, if she is at the end of her chain.



5. Suppose you bake a square cake, 10 inches on a side and 2 inches high. You frost it on the top and all four sides (but not the bottom). We want to split the cake among n people, and we want everyone to get equal shares of cake and frosting. Last time we figured out how to do it for $n = 2$, $n = 4$, and $n = 8$:



We had a number of other ideas too. What other numbers of people can you accommodate? Explain exactly how to cut the cake and why it is fair.

6. Celcius (°C) and Fahrenheit (°F) are two ways of measuring temperature. They are related by a linear equation.

   (a) Water freezes at 32°F and 0°C, and water boils at 212°F and 100°C. Use that information to write a formula for f, the temperature in Fahrenheit, in terms of c, the temperature in Celcius.

   (b) Imagine yourself in a cabin in the backwoods of northern Canada. You are very cold from a long day of skiing, and you need to know the temperature in Fahrenheit, so you can tell how much antifreeze to put into your car so that it will start in the morning. Unfortunately, this is Canada, so the only thermometer available is in Celcius, and you are too cold to multiply, so you can't use the formula you found in part (a). In the cabin with you are a roll of duct tape, a $33\frac{1}{3}$ RPM record player, a Beatles record from 1967, and a stopwatch, among other things. How can you compute the temperature in Fahrenheit without multiplying or dividing?

7. Let $f(x) = \ln(x)$. Sketch graphs of the following, without using a calculator:

$$f(x), \quad |f(x)|, \quad f(x+2), \quad f(3x-1), \quad e^{f(x)}$$

   Which of these functions has an inverse, without restricting its domain?

8. Suppose that $f(0) = 20$ and $f(3) = 40$.

   (a) Find a formula for $f(x)$, assuming f is linear.

   (b) Find a formula for $f(x)$, assuming f is exponential.

   (c) Find a formula for $f(x)$, assuming f is a quadratic polynomial. How many answers are there? If there are more than one, sketch as many as you can.