Douglass Houghton Workshop, Section 2, Thu 12/8/16

Worksheet So You Think You Know Calculus

1. A car speeds up at a constant rate from 10 to 70 mph over a period of half an hour, between $t = 0$ and $t = 1/2$. Its fuel efficiency, $E(v)$, measured in miles per gallon, depends on its speed, v, measured in miles per hour.

   (a) Write an integral for the total distance traveled by the car during the half hour.

   (b) Write an integral for the average fuel efficiency of the car during the half hour.

   (c) For speeds v greater than 70 mph suppose the relationship between E and v is given by
   $$E(v) = 2 + v^{-av}$$
   for some constant a. Using this formula, write an expression for the definition of the derivative $E'(82)$. Do not evaluate your expression.

2. (Fall, 2011) For positive A and B, the force between two atoms is a function of the distance, r, between them:
   $$f(r) = -\frac{A}{r^2} + \frac{B}{r^3}.$$

   (a) Find the zeroes of f in terms of A and B.

   (b) Find all critical points and inflection points of f in terms of A and B.

   (c) If f has a local minimum at $(1, -2)$ find the values of A and B. Using your values for A and B, justify that $(1, -2)$ is a local minimum.

3. (Fall, 2009) After an unusual winter storm, the EPA is concerned about potential contamination of a river. A new researcher has been assigned the task of taking a sample to test the water quality. She tried to get as close to the river as possible in her car, but was forced to park a feet away. She also cannot get closer to the lab by car. She needs to walk to the river, retrieve a water sample, and then walk the sample to a lab located $4a$ feet down the river and $2a$ feet from the river bank. If the researcher wants to walk as short a distance as possible, what path should she take as she walks from her car to the river and then from the river to the lab?

4. (Fall, 2014) Consider the family of functions given by
   $$f(x) = \frac{ax}{e^{0.5(bx)^2}}$$
   where a and b are constants with $a > 1$ and $b > 1$. Find all the global extrema of f on the interval $\left[\frac{1}{4b}, \infty\right)$, and classify them as local maxes or local mins.

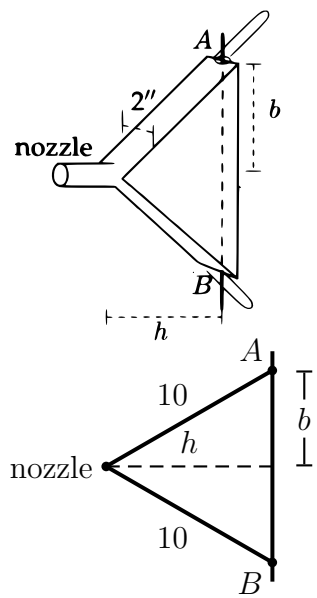
5. (Adapted from a Fall, 2004 Math 115 final) You spend New Year's Day eating apple pie while watching football on TV. The rate at which you eat pie is given by the function $r(t)$ where t is measured in hours and $r(t)$ is in liters/hour. Suppose $t = 0$ corresponds to 10 am.

   (a) Write a definite integral that represents the total amount of pie you consume between noon and 10 pm.

   (b) If your rate of pie eating is given by $r(t) = e^{-t} + 1$, use a left hand sum with three (3) subdivisions to estimate the amount of pie you eat in the first four hours of your binge.

   (c) Should your estimate in part (b) be an underestimate or an overestimate? Explain.

6. (This problem appeared on a Winter, 2008 Math 115 exam) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
   Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



7. (Fall, 2015) Consider the differentiable function Z defined by

$$Z(v) = \begin{cases} \dfrac{e^{v-1} - v}{(v-1)^2} & \text{if } v \neq 1 \\ \frac{1}{2} & \text{if } v = 1. \end{cases}$$

   Use the limit definition of the derivative to write an explicit expression for $Z'(1)$. *Your answer should not involve the letter Z.*