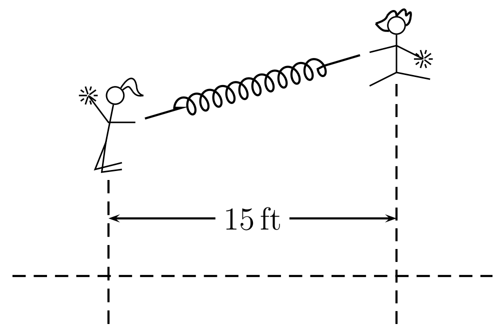
**Douglass Houghton Workshop, Section 2, Tue 12/06/16**

# Worksheet Romeo, Romeo, Wherefore Art Thou Romeo?

1. Jessica and Ajené are performing a routine in a STUNT cheerleading competition. They are both thrown straight up into the air from positions 15 feet apart, and they hold a slinky between them.




   (a) How fast is the slinky expanding when Ajené is 13.96 feet above the ground and rising at 1.6 ft/sec, while Jessica is 7 feet above the ground and falling at 14 ft/sec?

   (b) Jessica is thrown upward with an initial velocity of 18 ft/sec at time 0, and Ajené is thrown .3 seconds later with initial velocity 24 ft/sec. Both start 5 feet above the ground and are subject to the acceleration of gravity, which is $-32\,\text{ft/sec}^2$. Find formulas for their heights at time $t$.

   (c) Find the maximum and minimum length of the slinky between the time Ajené lifts off to the time Jessica is caught by her teammates.

2. (Fall 2008) This problem was a smorgasbord:

   (a) If $f(x)$ is even and $\int_{-2}^{2}(f(-x)-3)\,dx = 8$, find $\int_{0}^{2} f(x)\,dx$.

   (b) The average value of the function $g(x) = 10/x^2$ on the interval $[c, 2]$ is equal to 5. Find the value of $c$.

   (c) If people are buying UMAir Flight 123 tickets at a rate of $R(t)$ tickets/hour (where $t$ is measured in hours since noon on December 15, 2008), explain in words what $\int_{3}^{27} R(t)\,dt$ means in this context.

   (d) Suppose that the function $N = f(t)$ represents the total number of students who have turned in this exam $t$ minutes after the beginning of the exam. Interpret $(f^{-1})'(325) = 2$.

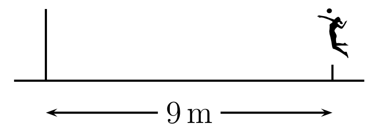
   (e) Find $k$ so that the function $h(x)$ below is continuous for all $x$.

$$h(x) = \begin{cases} x^2 - 1 & \text{if } x \le 1 \\ 6\sin(\pi(x-0.5)) + k & \text{if } x > 1 \end{cases}$$

3. (This problem appeared on a Fall, 2005 Math 115 Final Exam) Using techniques from calculus, find the dimensions which will maximize the surface area of a circular cylinder whose height $h$ and radius $r$, each in centimeters, are related by

$$h = 8 - \frac{r^2}{3}.$$

4. At critical moment in this weekend's NCAA tournament, it is Rocky's turn to serve. She stands at the service line, which is 9 m from the net. She hits the ball at height $h$, and gives it an initial velocity of $v_0$ at an initial upward angle of $\theta$.




(a) The horizontal velocity of the ball is $v_0 \cos\theta$. How long will it take the ball to get to the net?

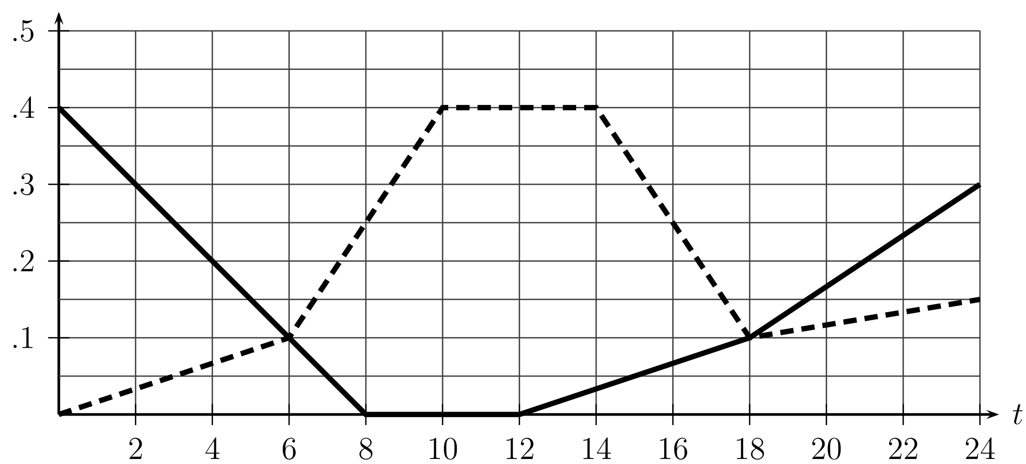
(b) The initial vertical velocity is $v_0 \sin\theta$. If acceleration due to gravity is $-9.8\,\mathrm{m/sec}^2$, find a formula for the ball's vertical velocity at time $t$.

(c) Now find the ball's height at time $t$.

(d) Find an inequality describing which values of $v_0$, $h$, and $\theta$ will make the ball clear the net, which is 2.24 m high.

(e) The other team's court is also 9 m long. Find another inequality that ensures that the ball will land inside that court.

5. (Fall, 2011) Suppose the graph below shows the rate of snow melt and snowfall on Mount Arvon, the highest peak in Michigan (at a towering 1970 ft), during a day (24 hour period) in April of last year. The function $m(t)$ (the solid curve) is the rate of snow melt, in inches per hour, $t$ hours after the beginning of the day. The function $p(t)$ (the dashed curve) is the snowfall rate in inches per hour $t$ hours after beginning of the day. There were 18 inches of snow on the ground at the beginning of the day.




(a) Over what period(s) was the snowfall greater than the snow melt?

(b) When was the amount of snow on Mount Arvon increasing the fastest? When was it decreasing the fastest?

(c) When was the amount of snow on Mount Arvon the greatest? Explain.

(d) How much snow was there on Mount Arvon at the end of the day (at $t = 24$)?

(e) Sketch a well-labeled graph of $P(t)$, an antiderivative of $p(t)$ satisfying $P(0) = 0$. Label and give the coordinates of the points on the graph of $P(t)$ at $t = 10$ and $t = 18$.