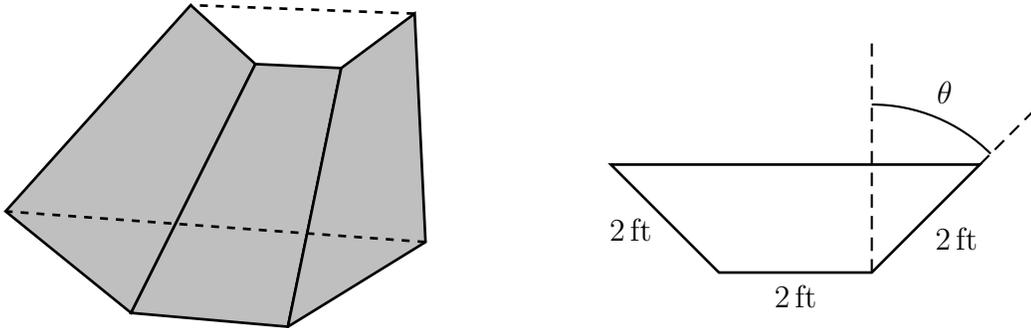


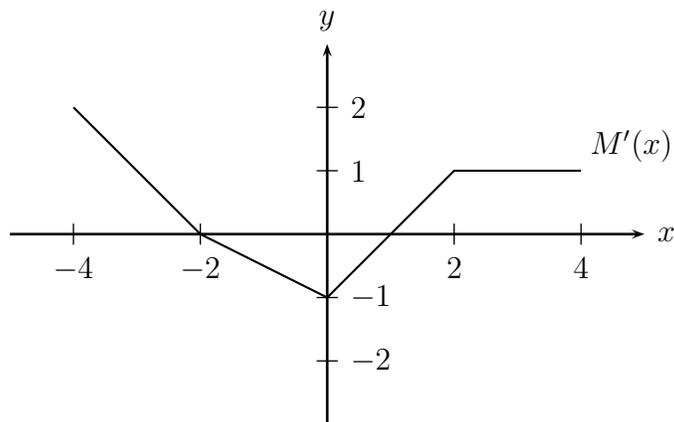
Douglass Houghton Workshop, Section 2, Thu 12/01/16
Worksheet Quoth the Raven, "Nevermore"

1. A trough, as shown below, is to be made with a base that is 2 feet wide and 10 feet long. The sides of the trough are also 2 feet wide by 10 feet long, and are to be placed so they make an angle θ with the vertical.

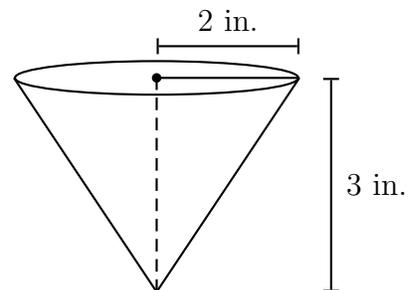


- (a) What is the area, in terms of θ , of a cross section of the trough perpendicular to its long side? What is the volume of the trough?
- (b) What angle θ will give the trough the largest volume, and what is that volume? [Hint: you can always replace $\cos^2(\theta)$ with $1 - \sin^2(\theta)$.]

2. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.



3. (This problem appeared on the Fall, 2008 Math 115 Final Exam) Suppose that you are brewing coffee and that hot water is passing through a special, cone-shaped filter. Assume that the height of the conic filter is 3 in. and that the radius of the base of the cone is 2 in. If the water is flowing out of the bottom of the filter at a rate of $1.5 \text{ in}^3/\text{min}$ when the remaining water in the filter is 2 in. deep, how fast is the depth of the water changing at that instant?



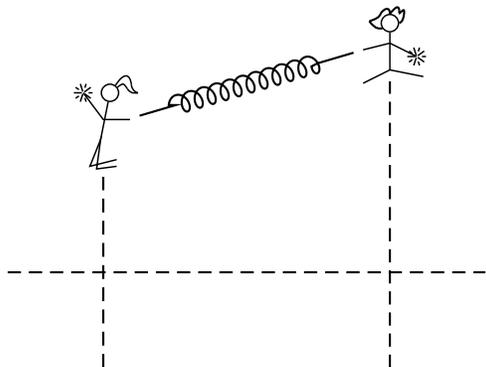
4. (This problem appeared on the Winter, 2015 Math 115 Final Exam) For nonzero constants a and b with $b > 0$, consider the family of functions given by

$$f(x) = e^{ax} - bx.$$

Note that the derivative and the second derivative of $f(x)$ are given by

$$f'(x) = ae^{ax} - b \quad \text{and} \quad f''(x) = a^2e^{ax}.$$

- (a) Suppose the values of a and b are such that $f(x)$ has at least one critical point. For the domain $(-\infty, \infty)$, find all critical points of $f(x)$, all values of x at which $f(x)$ has a local extremum, and all values of x at which $f(x)$ has an inflection point. (Note that your answer(s) may include the constants a and/or b .)
- (b) Which of the following conditions on the constant a guarantee(s) that $f(x)$ has at least one critical point in its domain $(-\infty, \infty)$?
- (i) $a < 0$ (ii) $0 < a < b$ (iii) $b < a$
- (c) Find exact values of a and b so that $f(x)$ has a critical point at $(1, 0)$.



5. Ajené and Jessica are performing a routine in a STUNT cheerleading competition. They are both thrown straight up into the air from positions 15 feet apart, and they hold a slinky between them.

- (a) How fast is the slinky expanding when Ajené is 13.96 feet above the ground and rising at 1.6 ft/sec, while Jessica is 7 feet above the ground and falling at 14 ft/sec?
- (b) Jessica is thrown upward with an initial velocity of 18 ft/sec at time 0, and Ajené is thrown .3 seconds later with initial velocity 24 ft/sec. Both start 5 feet above the ground and are subject to the acceleration of gravity, which is -32 ft/sec^2 . Find formulas for their heights at time t .
- (c) Find the maximum and minimum length of the slinky between the time Ajené lifts off to the time Jessica is caught by her teammates.