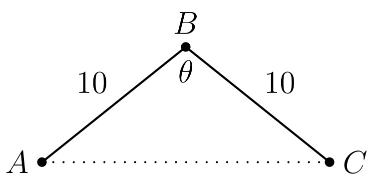
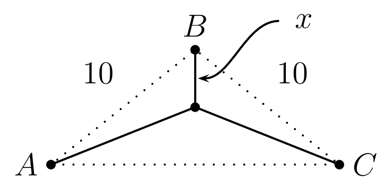
Douglass Houghton Workshop, Section 2, Tue 11/17/16

Worksheet Now is the Winter of our Discontent

1. Shortest Network. So far we've looked at the case where the cities are at the corners of an isosceles triangle like the one below. We know:



- When angle B is 70° or 90°, it is possible to improve upon the Λ-shaped network shown by building a roundabout south of B and connecting it to all three cities.
- However, when B is 150°, the Λ is better than all possible ⅄'s.



   (a) Suppose the measure of angle B is θ. Use the law of cosines to write a formula for the ⅄-shaped network to the right.

   (b) Call that function $L_\theta(x)$. Put your calculator in degrees mode and plot $L_{70}(x)$, $L_{90}(x)$, and $L_{150}(x)$, for x from 0 to 10. Put the graphs on the board.

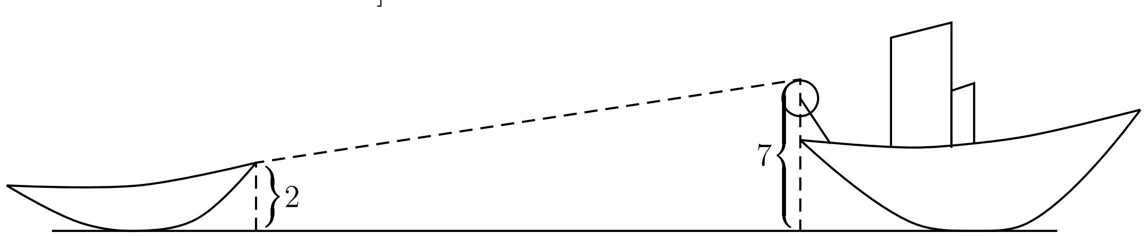
   (c) The shapes seem about the same. What can you see in the graphs that explains the fact that in two cases the Λ can be improved, and in the other it can't? (Remember the Λ is $x = 0$.)

   (d) Use calculus to figure out which Λ's can be improved, and which can't. State the result in the form: "Any Λ-shaped network with an angle smaller than ______ can be improved".

   (e) This is for those who like to compute and simplify. Show that the function $L_\theta(x)$ defined above is always concave up, by finding and simplifying its second derivative.

2. (Adapted from a Fall, 2006 Math 115 Final Exam.) Rocky is sailing in San Juan harbor. Unfortunately the boat has run out of gas, so Rocky hails a passing tugboat to give her a tow.

   The tugboat captain throws Rocky a line, and she attaches it to her boat 2 meters above the water line. The other end of the cable is attached to a wheel of radius 0.5 meters sitting on the back of the tugboat. The top of the wheel is 7 meters above the water, and turns at a constant rate of 1 revolution per second. [See the figure below—not drawn to scale.]



(a) At what rate is the length of the cable between the two boats changing?

(b) How fast is Rocky's boat being pulled forward when it is 10 meters away from the tugboat?

3. (Adapted from a Fall, 2001 Math 115 final exam) Ngan starts a business selling custom T-Shirts. Her cost and revenue functions (in dollars) are

$$C(q) = 400 + 8q \qquad \text{and} \qquad R(q) = 60q^{.75}$$

where q is the number of T-Shirts Ngan produces.

(a) What is the product's fixed cost?

(b) Last year, Ngan produced 2400 units. What was her profit?

(c) Find formulas for the marginal cost and marginal revenue, and evaluate at $q = 2400$.

(d) Ngan wants to increase production and do better this year. Based on the marginal cost and marginal revenue *at this point* ($q = 2400$), explain whether her strategy is sound.

4. November is National Novel Writing Month, and many people around the country attempt to complete a first draft of a novel in the course of the month. One of them is Mark's friend Chris. At the end of every day in November she uploads her manuscript to a website (`http://tinyurl.com/worlds-over-again`), which counts how many words she has written. Here are her counts from last year, rounded to the nearest hundred:

Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count	Nov.	Count
1	1700	6	10400	11	17000	16	26100	21	31700
2	3400	7	12100	12	18000	17	26600	22	33100
3	5400	8	13400	13	19000	18	26600	23	35200
4	6500	9	13600	14	20000	19	26600	35200	
5	8700	10	15300	15	25000	20	28700		

(a) Let x be the time in days since the start of November, and let $W(x)$ be the total number of words Chris has written at time x. (So, for instance, $W(14) = 20000$.) Assume that each day Chris writes at a steady rate, from midnight to midnight. (But different rates for different days.) Draw a graph of $W(x)$ for last week (x from 14 to 21).

(b) Let $w(x)$ be the derivative of $W(x)$. Draw a graph of $w(x)$ for x from 14 to 21.

(c) Now consider the function $F(t)$, which is the area between the line $x = 14$, the line $x = t$, the x-axis, and the graph of $w(x)$. Make a table of values showing $F(14), F(15), \ldots, F(21)$. What do you notice? Explain this result.