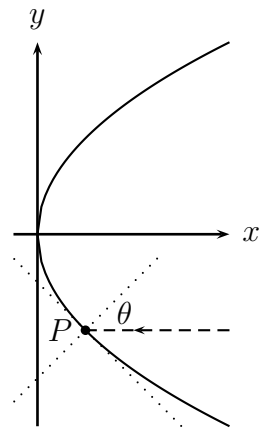
Douglass Houghton Workshop, Section 2, Tue 11/1/16

Worksheet Labradoodle

1. Last time we thought about a parabolic mirror in the shape of the graph of $y = \pm\sqrt{4x}$. So far we've found:



   - A light ray $y = -b$ hits the mirror at $P = (b^2/4, -b)$.
   - The slope of the tangent at that point is $-2/b$.
   - The normal line at the same point has slope $b/2$.
   - When a line makes an angle θ with the x-axis, it has slope $\tan\theta$.
   - So if we call the angle between the light ray and the normal line θ, then $\tan(\theta) = b/2$.
   - $\tan(2x) = 2\tan x/(1 - \tan^2 x)$

   (a) Draw the picture on the board.

   (b) To the ray, the mirror looks flat, just like the tangent line. Draw the reflected ray. What angle does it make with the x-axis?

   (c) What is the slope of the reflected ray?

   (d) Write an equation for the reflected ray.

   (e) Where does the reflected ray intersect the x-axis? What is surprising about this answer?

   (f) Graph several rays, with their reflections.

   (g) What's cool about this type of mirror?

2. In "The 12 days of Christmas", a certain poultry-afficianado receives a number of gifts from her true love:

   Day 1: A partridge in a pear tree. How to get it down?

   Day 2: 2 turtle doves, and another partridge in a pear tree. Is it the same tree?

   Day 3: 3 French hens, 2 more turtle doves, and another partidge.

   ...

   Day 12: 12 drummers drumming (loudly), eleven pipers piping (make them stop!), ..., and yet another partridge in a pear tree.

   (a) If item 1 is "partridge", item 2 is "turtle dove", etc., then write a formula for the total number of item n's received.

   (b) Of which item does Mr. Truelove send the most? (Solve using calculus.)

3. The three cities in the pictures below are at the corners of a 45°-45°-90° triangle whose legs are 50 miles long. The three mayors, working together, would like to build roads between them in such a way that there is a way to get from any one city to any other city.

(Say, A is Ann Arbor, B is Flint, and C is Port Huron.) The first, simple proposal (on the left) is to build a road from A to B and another from B to C. That would certainly work. But roads are expensive, and one of the mayors (who, luckily, studied calculus) proposes building roads from A and C to a point D just south of B, then building a road north from there to B.

(a) Let x be the length of the north-south road in the second proposal. What does it mean if $x = 0$?

(b) Calculate the total length of the new network in terms of x. Hint: "Law of cosines".

(c) Can you find a value of x which will produce a shorter network than the simple proposal?

4. Alec drives east on the Ohio Turnpike to a Super Smash Bros. Game Cube tournament He takes a ticket out of the machine in Toledo, and then turns it in at the toll booth near Columbus. Along with his change, the State Trooper in the toll booth hands Alec a speeding citation, and says that he *knows* Alec was going exactly 70 mph at some point on his trip. How does the Mean Value Theorem tell the trooper that?

5. (This problem appeared on the Fall, 2008 Math 115 Final Exam) At the Michigan-Ohio State basketball game this year, the Michigan Band discovers that the amount of time it spends playing "Hail to the Victors" has a direct impact on the number of points our team scores. If the band plays for x minutes, then the Wolverines will score

$$W(x) = -.48x^2 + 7.2x + 63$$

points. Assume that the band can play for a maximum of 10 minutes.

(a) How long should the band play to maximize the number of points Michigan scores?

(b) The band affects how many points Ohio State scores as well. x minutes of playing results in the Buckeyes scoring

$$B(x) = -x^2 + 8x + 84$$

points. Find the number of minutes the band should play to maximize the margin of victory for Michigan.