Douglass Houghton Workshop, Section 2, Tue 9/20/16

Worksheet Eat and Be Merry

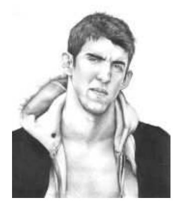
1. *Michael Phelps: The Sequel* Michael Phelps took all the money (let's say it's 2 million dollars) he got for endorsing Speedo, Visa, Subway, Frosted Flakes, and Head & Shoulders shampoo, and put it into a bank. The bank has several accounts available. For each, write an expression for how much Michael will have t years from now.

   (a) 6% interest, compounded annually.

   (b) 5% interest, compounded monthly.

   (c) 4% interest, compounded daily.

   (d) interest rate r, compounded n times per year.

   The bank also has something called "continuously compounded interest", which means that the number of compoundings per year is really really large. Write a limit expression for the amount of money he'll have if he gets interest rate r, compounded continuously.

2. Bankers and financial advisors use what they call the **Rule of 70**. It says:

   > If you invest money at annual interest rate r percent, it will take about $70/r$ years for your money to double.

   (So, for instance, \$500 invested at 5% interest will be worth \$1000 in about about 14 years, because $14 = 70/5$.)

   (a) Explain why the Rule of 70 works, and what assumptions you need to make it work. Hint: recall what we learned from Michael Phelps's towel:

   $$\lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n = e^r.$$

   (b) Devise a similar rule for the time it takes your money to triple at r% interest.

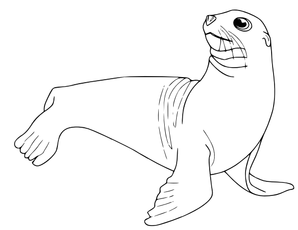
3. (This problem appeared on a Fall, 2010 Math 115 exam.) Before the industrial era, the carbon dioxide (CO_2) level in the air in Ann Arbor was relatively stable with small seasonal fluctuations caused by plants absorbing CO_2 and producing oxygen in its place. Typically, on March 1, the CO_2 concentration reached a high of 270 parts per million (ppm), and on September 1, the concentration was at a low of 262 ppm. Let $G(t)$ be the CO_2 level t months after January 1.

   (a) Assuming that $G(t)$ is periodic and sinusoidal, sketch a neat, well-labeled graph of G with $t = 0$ corresponding to January 1.

   (b) Determine an explicit expression for G, corresponding to your sinusoidal graph above.

4. We've all seen 6-sided dice, and we presume they are "fair", in the sense that all 6 sides are equally likely to land on the bottom. Can you construct a fair 4-sided die? How about an 8-sided die? What other sizes are possible?

5. Jade is studying a population of sea lions in California. Suppose that the population changes according to the rule:



$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2016, $P(1)$ is the population 1 year later, etc.

   (a) Make up a (short) story about sea lions that yields that formula as the result.

   (b) Suppose there are 320 sea lions in 2016. What will happen in the long run?

   (c) Suppose instead that there are 800 sea lions in 2016. Now what happens?

   (d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?

   (e) Explain these results pictorially by drawing the graphs of $y = x$ and $y = 1.5x - 200$. Start at $(200, 200)$, go down to the other graph, and then over to $y = x$. That's the new population. Repeat. Then start at 800.

6. Repeat the last problem, but for the rule

$$P(n+1) = .75P(n) + 200.$$

7. A population equilibrium is **stable** if the population moves toward the equilibrium, rather than away from it. Which of the last two sea lion scenarios has a stable equilibrium?

8. Consider the double Ferris wheel: `http://www.youtube.com/watch?v=2DV4hN0c8WU`

   (a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

   (b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

   (c) Suppose Erika is seated at one end of the big arm, i.e., at the center of one of the small wheels. She is playing the tuba. Suppose she starts as far to the right as possible. Write a formula for her height t seconds after the wheel starts, relative to the center of the big wheel.

   (d) Do the same for Erika's horizontal position.

   (e) Now suppose Addison is in a seat on one of the small wheels. She is playing the trumpet. Write formulas for Addison's x and y position *relative to Erika*.

   (f) Now find formulas for Addison's position relative to the center of the big wheel.