Douglass Houghton Workshop, Section 2, Thu 9/15/16

Worksheet Dynamite

1. *The Saga of Michael Phelps: Conclusion* Last time we found that Michael Phelps can always make himself dryer by splitting his towel, but there's a limit to how dry he can get. In particular, here are the numbers for not splitting the towel at all and for splitting it into 10,000 pieces:

Towel Size	.25	.5	1	2	3	4
wetness (1 piece)	0.8000	0.6667	0.5000	0.3333	0.2500	0.2000
wetness (10,000 pieces)	0.7788	0.6065	0.3679	0.1354	0.0498	0.0183

Cutting into more than 10,000 pieces doesn't seem to make much difference. So for each towel T, there is a wetness $N(T)$ after normal toweling, and there seems to be a "magic number" $M(T)$, which is the *limit* to how dry Michael can get by splitting the towel.

   (a) Make a graph with towel size on the x-axis and wetness on the y-axis. Plot the points you have for $N(T)$, the result of normal toweling, and $M(T)$, the result of split towelling.

   (b) What's the formula for $N(T)$? (We found this a couple of days ago).

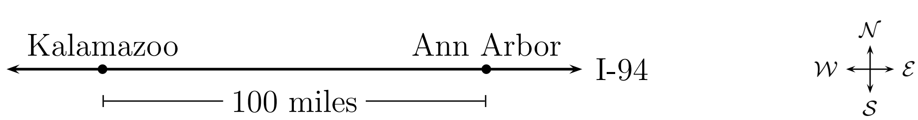
   (c) What kind of function does $M(T)$ look like? Hint: Compare $M(1)$ with $M(2)$.

   (d) Verify your guess by finding a formula that fits the data.

   (e) Using the formula we found on Tuesday for splitting the towel into n parts, write a limit equation to express the result in part (d).

2. Kalamazoo is 100 miles west of Ann Arbor along Route 94. Suppose we know the temperature at every point on the road between the two cities, and we express that information as

$$T(x) = \text{ the temperature in Fahrenheit at a point } x \text{ miles west of Ann Arbor.}$$



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

3. dBase™ was a database management system popular on IBM PCs back in the 80s. It included a programming language with limited capabilities; for instance, there was no command in the language to take the square root of a number. There were, however, two functions called `LOG`(x) and `EXP`(x) which produced $\ln(x)$ and e^x, respectively. How could you use them to produce $\sqrt{x}$?

4. Consider the double Ferris wheel: `http://www.youtube.com/watch?v=2DV4hNOc8WU`

   (a) Use a watch to estimate the periods of the large rotation and the smaller rotation.

   (b) Estimate the radii of the two rotations, knowing as you do that the seats are designed for humans.

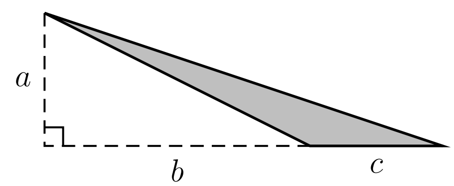
   (c) Suppose Erika is seated at one end of the big arm, i.e., at the center of one of the small wheels. She is playing the tuba. Suppose she starts as far to the right as possible. Write a formula for her height t seconds after the wheel starts, relative to the center of the big wheel.

   (d) Do the same for Erika's horizontal position.

   (e) Now suppose Addison is in a seat on one of the small wheels. She is playing the trumpet. Write formulas for Addison's x and y position *relative to Erika*.

   (f) Now find formulas for Addison's position relative to the center of the big wheel.

5. Find the area of the shaded triangle:



6. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a yellow cake with chocolate frosting. It's getting a bit drippy while we decide how to cut it.

   We have a solution that seems to work, at least for some numbers of people. So we should

   (a) Try to prove that it always works,

   (b) Find a way to make each person's pieces adjacent, to reduce cutting as much as possible, and

   (c) Simplify the cuts needed as much as possible.

7. (This problem appeared on a Winter, 2016 Math 115 exam) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} xe^{Ax} & \text{if } x < 3 \\ C(x-3)^2 & \text{if } 3 \le x \le 5 \\ \frac{130}{x} & \text{if } x > 5 \end{cases}$$

   Suppose that $f(x)$ is continuous at $x = 3$, $\lim_{x\to 5^+} f(x) = 2 + \lim_{x\to 5^-} f(x)$, and $\lim_{x\to\infty} f(x) = -4$. Find A, B, and C.