Douglass Houghton Workshop, Section 2, Tue 9/13/16

Worksheet Chocolate Frosting

1. As we know, Breanna has been to 43 states. As a result, she began collecting state memorabilia. She got a 50-states lunchbox, a New York Thruway Bell, a California Redwood coaster, and more. This year, after she had accumulated 40 state-related items, she had had enough and wanted to move on to something else. But her (well-meaning) friends and relatives, observing the state swag around her, assumed she wanted more, and started giving her things.

   Write formulas for the number of state-themed items Breanna will have t years from now, under the following conditions:

   (a) Breanna receives 5 new state items every year.

   (b) In year t Breanna receives one new state item for each pair of items she had in year $t-1$.

   (c) Breanna receives 1 state item next year, 2 the year after that, 3 the year after that, etc.

2. Last time we found formulas for Michael Phelps' dampness after regular towelling and split towelling. Assuming Michael's body is $1\,\mathrm{m}^2$, the towel is $T\,\mathrm{m}^2$, and he starts with 1 liter of water on him, we have

$$\begin{array}{l}\text{wetness after}\\ \text{regular toweling}\end{array} = \frac{1}{1+T} \qquad \begin{array}{l}\text{wetness after}\\ \text{“split” toweling}\end{array} = \frac{1}{(1+T/2)^2}.$$

   Let's see just how much this "splitting" idea will buy us.

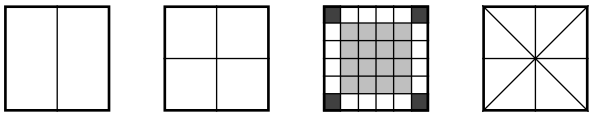
   (a) Suppose Michael splits his towel into 3 parts, and uses all three. How wet will he be? How about if he splits it into n parts?

   (b) Use calculators to fill in the table below with 4-decimal place numbers.

T	$n=1$	$n=10$	$n=100$	$n=1000$	$n=10000$
$1\,\mathrm{m}^2$					
$2\,\mathrm{m}^2$					
$4\,\mathrm{m}^2$					
$\frac{1}{2}\,\mathrm{m}^2$					

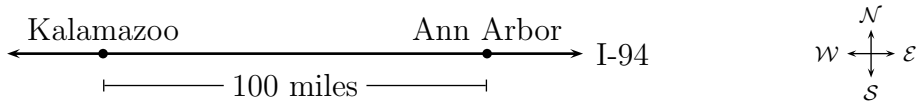
   (c) Consider the $1\,\mathrm{m}^2$ towel. How would you describe the effect of dividing it into more and more pieces? For instance, does dividing more always make Michael dryer? Might it make him wetter? Can he get as dry as he might possibly want with that one towel? Does it even matter how big his towel is?

3. So we still have this square cake, 10 inches on a side and 2 inches high, frosted on the top and all four sides. It is a yellow cake with chocolate frosting. It's getting a bit drippy while we decide how to cut it. Last week we figured out how to do it for $n = 2$, 4, 6, and 8 people.



We had several other ideas too. What other numbers of people can we handle?

4. Kalamazoo is 100 miles west of Ann Arbor along Route 94. Let $T(x)$ be the temperature in Fahrenheit at a point x miles west of Ann Arbor.



   (a) Define a function A in terms of T so that $A(m)$ is the temperature in Fahrenheit at a point m miles east of Kalamazoo.

   (b) Define a function B in terms of T so that $B(k)$ is the temperature in Fahrenheit at a point k **kilometers** east of Kalamazoo. (1 mile = 1.6 kilometers.)

   (c) Define a function C in terms of T so that $C(k)$ is the temperature in **Celcius** at a point k kilometers east of Kalamazoo.

5. (This problem appeared on a Fall, 2012 Math 115 exam) Suppose p represents the price of a reuben sandwich at a certain restaurant on State Street. $R(p)$ represents the number of reubens the restaurant will sell in a day if they charge $\$p$ per reuben.

   (a) What does $R(5.5)$ represent in the context of this situation?

   (b) Assuming R is invertible, what does $R^{-1}(305)$ represent?

   (c) The owner of the restaurant also has a Church Street location. It doesn't get quite as much business, and the owner finds that the State Street store sells 35% more reubens than the Church Street store sells at the same price. Let $C(p)$ be the number of reubens the Church Sreet location sells in a day at a price of $\$p$ each. Write a formula for $C(p)$ in terms of $R(p)$.

   (d) The owner starts doing research on reuben sales at the State Street location; he wants to know how the number of reubens sold is related to price. He finds that every time he raises the price by \$1 per reuben, the number sold in a day decreases by 20%. Let the constant B represent the number of reubens sold in a day at the State Street store if the price of reubens is \$5 each. Write a formula for $R(p)$ involving the constant B. Assume the domain of R is $1 \leq p \leq 25$.